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...OSOPHY.

...SITIONS IN

...DROSTATICS

...O ARE NOT CANDIDATES FOR
...FOR THE DEGREE OF B.A.

CAMBRIDGE:
...PRINTED BY JOHN W. PARKER, UNIVERSITY PRINTER;
...SON, AND GRANT, CAMBRIDGE;
H. BAILY & Co., LONDON.
M.DCCC.XXXVIII.

THE

CAMBRIDGE COURSE

OF

ELEMENTARY NATURAL PHILOSOPHY.

BEING THE DEMONSTRATIONS OF THE PROPOSITIONS IN

MECHANICS AND HYDROSTATICS

IN WHICH THOSE PERSONS WHO ARE NOT CANDIDATES FOR HONOURS ARE EXAMINED FOR THE DEGREE OF B.A.

CAMBRIDGE:
PRINTED BY JOHN W. PARKER, UNIVERSITY PRINTER;
STEVENSON, AND GRANT, CAMBRIDGE;
A. H. BAILY & Co., LONDON.
M.DCCC.XXXVIII.

PREFACE.

THE following pages contain the demonstrations of the questions in the Course of Elementary Natural Philosophy in which the University of Cambridge requires the Candidates for the Degree of B.A. in 1841 and succeeding years to be prepared for Examination. The Enunciations adopted by the Senate have here been rigidly adhered to, and the Propositions are proved in the same order as they have been numbered by Authority. The writer did not consider himself at liberty to alter, or in any respect to deviate from, the scheme approved of by the Senate, conceiving that the Examiners to be appointed under the new system will not themselves possess the power of setting any other questions than those sanctioned by the public expression of the will of the University.

It has been endeavoured to give such proofs and explanations as might be found intelligible and easy by students possessing a moderate acquaintance with Arithmetic, the first principles of Algebra, and Euclid's Elements. Strict and brief demonstrations have been preferred to long diffuse explanations and illustrations, from the idea that the former were the

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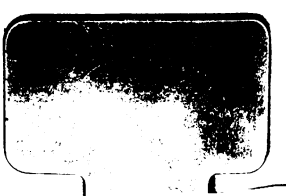

ADVERTISEMENT.

THE subjects in which those persons who are not Candidates for Honours are examined for the degree of B.A. are,

I. The Acts of the Apostles in the original Greek.

II. One Greek Classic; appointed in the last week of the Lent Term in every year for the year next but one following. Thus the Classical subjects for the Examinations of 1841 will be fixed on in the last week of the Lent Term of 1839.

III. One Latin Classic; appointed in the same manner as the last subject.

IV. Paley's Moral Philosophy.

V. Such Mathematical Subjects as are contained in the following Schedule*.

* The Examiners are required to publish the names of the persons whom their respective Colleges permit to offer themselves for Examination at the Bachelors' Commencement, in a list, which is to be arranged in alphabetical order, and separated into two Divisions. The distribution of the different subjects, and the times of Examination are fixed by the the following Table.

	Div.	9 to 12.	Div.	12½ to 3½.
Wednesday.	1	Acts of the Apostles.....	2	Greek Subject.
Thursday	1	Greek Subject.....	2	Acts of the Apostles.
Friday.....	1	Euclid.....	2	Latin Subject.
Saturday	1	Latin Subject.....	2	Euclid.
Monday	1	{ Mechanics and Hydrostatics }	2	Moral Philosophy.
Tuesday	1	Moral Philosophy.....	2	{ Mechanics and Hydrostatics.
Wednesday...	1	Arithmetic and Algebra..	2	Arithmetic and Algebra.

Schedule of Mathematical Subjects of Examination, for the Degree of B.A. of Persons not Candidates for Honours.

Arithmetic.

Addition, subtraction, multiplication, division, reduction, rule of three; the same rules in vulgar and decimal fractions; practice, simple and compound interest, discount, extraction of square and cube roots, duodecimals.

Algebra.

1. Definitions and explanation of algebraical signs and terms.
2. Addition, subtraction, multiplication and division of simple algebraical quantities and simple algebraical fractions.
3. Algebraical definitions of ratio and proportion.
4. If $a : b :: c : d$ then $ad = bc$, and the converse:
   also $b : a :: d : c$,
   and $a : c :: b : d$,
   and $a+b : b :: c+d : d$.
5. If $a : b :: c : d$,
   and $c : d :: e : f$,
   then $a : b :: e : f$.
6. If $a : b :: c : d$,
   and $b : e :: d : f$,
   then $a : e :: c : f$.
7. Geometrical definition of proportion. (Euc. Book v. Def. 5.)
8. If quantities be proportional according to the algebraical definition, they are proportional according to the geometrical definition.
9. Definition of a quantity *varying as* another, *directly* or *inversely*, or as two others *jointly*.

Euclid.

Books i. ii. iii;

Book vi. Props. 1. 2. 3. 4. 5. 6.

[The remaining portion of the Schedule forms the Table of Contents of this Book.]

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MECHANICS.

Chapter I.

ARTS.
1—15. Definitions of Force, Weight, Quantity of Matter, Density, Measure of Force.

Chapter II. *The Lever.*

ARTS.
17. Definition of Lever.

18. Axioms.

19. Prop. 1. A horizontal prism or cylinder of uniform density will produce the same effect by its weight as if it were collected at its middle point.

20. Prop. 2. If two weights acting perpendicularly on a straight lever on opposite sides of the fulcrum balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to their sum.

21. Prop. 3. If two forces acting perpendicularly on a straight lever in opposite directions and on the same side of the fulcrum balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to the difference of the forces.

23. Prop. 4. To explain the different kinds of levers.

24. Prop. 5. If two forces acting perpendicularly at the extremities of the arms of any lever balance each other, they are inversely as the arms.

25. Prop. 6. If two forces acting at any angles on the arms of any lever balance each other, they are inversely as the perpendiculars drawn from the fulcrum to the directions in which the forces act.

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32. Definition of Pulley.

32. Prop. 11. In the single moveable pulley where the strings are parallel, there is an equilibrium when the power is to the weight as 1 to 2.

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34. Prop. 13. In a system in which each pulley hangs by a separate string and the strings are parallel, there is an equilibrium when $P : W :: 1$: that power of 2 whose index is the number of moveable pulleys.

35. Prop. 14. The weight (W) being on an inclined plane and the force (P) acting parallel to the plane, there is an equilibrium when $P : W ::$ the height of the plane : its length.

36. Definition of Velocity.

37. Prop. 15. Assuming that the arcs which subtend equal angles at the centers of two circles are as the radii of the circles, to shew that if P and W balance each other on the wheel and axle, and the whole be put in motion, $P : W ::$ W's velocity : P's velocity.

38. Prop. 16. To shew that if P and W balance each other in the machines described in Propositions 11, 12, 13 and 14, and the whole be put in motion, $P : W ::$ W's velocity in the direction of gravity : P's velocity.

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ARTS.

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43. Prop. 19. To find the center of gravity of any number of heavy points; and to shew that the pressure at the center of gravity is equal to the sum of the weights in all positions.

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ARTS.
13. Definition of Specific Gravity.

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15. Prop. 8. When a body of uniform density floats on a fluid, the part immersed : the whole body :: the specific gravity of the body : the specific gravity of the fluid.

16. Prop. 9. When a body is immersed in a fluid, the weight lost : whole weight of the body :: the specific gravity of the fluid : the specific gravity of the body.

18. Prop. 10. To describe the *hydrostatic balance*, and to shew how to find the specific gravity of a body by means of it, 1st, when its specific gravity is greater than that of the fluid in which it is weighed; 2dly, when it is less.

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28. Prop. 19. To describe the construction of the common *pump* and its operation.

ARTS.
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THE ELEMENTS

OF

MECHANICS AND HYDROSTATICS.

THE few explanatory articles and paragraphs that are enclosed in brackets form no part of the Course required to be read by the Candidate for the degree of B.A. *Those in smaller type are illustrative of the Definitions and Propositions which they immediately follow. The whole of the extra matter so introduced forms but a very small, and a very easy, portion of the book; and they who come to this subject for the first time will do well not to omit it.*

MECHANICS.

CHAPTER I.

DEFINITIONS; EXPLANATION OF STATICAL FORCES; THE MANNER IN WHICH THEY ARE MEASURED, AND REPRESENTED.

1. THE science of MECHANICS investigates the causes that prevent or produce motion in bodies, or that tend to produce or prevent motion.

It is divided into two parts. The one, which investigates the conditions fulfilled when a body is in a state of rest, is called STATICS. The other, which treats of the causes and the effects of motion, is called DYNAMICS.

Thus it is the province of STATICS to shew how the roof of a building is supported by the cross beams and the walls. If the roof give way and fall, it would belong to DYNAMICS to account for the circumstances attending the fall;—to explain why the motion takes place in one direction rather than in another; to determine the time elapsed in falling, and the swiftness of motion at any instant.

The part of MECHANICS treated on in the following pages is THE ELEMENTS OF STATICS.

2. DEF. Whatever the cause may be that produces or prevents motion, or that tends to produce or prevent motion, it is called a FORCE.

[If a heavy body, as stone, be laid on the open hand, experience informs us that to prevent the stone from falling the hand must make some effort. Again, to set a ball rolling along the ground requires some exertion. The effort, or exertion, is called in either case a FORCE; and though the effect be not great enough to prevent entirely the fall of the stone, or to communicate motion to the ball, yet it is still denominated a FORCE.

From the definition of STATICS given in Art. 1 it will readily be understood that in that branch of MECHANICS the conditions are investigated, which are fulfilled by those forces only that keep a body at rest.]

3. DEFS. (1) All bodies we are acquainted with, if left to themselves, would fall to the earth's surface. This power residing in the earth of drawing all substances to its surface is called THE FORCE OF GRAVITY.

(2) The act of a body's pressing downwards when laid on any substance is called the GRAVITATION of the body.

(3) The precise amount of gravitation residing in any particular body is termed its WEIGHT.

4. [*The* WEIGHTS *of bodies may be thus compared.* If two bodies, when successively attached in the same manner to a spring, so that they may act upon it by their weights in the same way, produce the same effects,—(by bending the spring to the same extent),—the weights of the bodies will be equal. Any other body that produces the same effect on the spring by its weight as both the former bodies when applied at the same time do by their weights, is double the weight of either. And by means of such a contrivance as this spring, bodies might be shewn to be three, four, or any number of times the weight of the first body.

The WEIGHT of any body is thus *measured.* The weight of a certain bulk of some particular substance is first fixed upon as a standard. Thus the weight of a piece of *lead* of a certain size is called a *pound,* and then any other body whose gravitation produces the same effect as four, or six, or ten such pieces of lead, is four or six or ten pounds in weight, as the case may be.]

5. DEF. The substance, or material, of which any body is formed, is called MATTER.

[As all bodies we are acquainted with have Weight, the property of having weight is to be considered as necessarily and inherently belonging to all matter; and it is with respect to its property of having weight that matter is made the subject of mathematical investigation. The existence of matter, under whatever form it may appear, is therefore tested by that form having weight; and in the same ratio or degree that one body has more weight than another, it is concluded that the former body contains more Matter than the latter does.]

6. DEF. THE QUANTITIES OF MATTER that different bodies contain are proportional to the weights of the bodies.

[Thus, if a body *A* weigh *one* pound, and another body *B* weigh *three* pounds, the quantity of matter contained in *A* is said to be to the quantity contained in *B* as 1 to 3; or *B* is said to contain three times as much matter as *A* does.

The exact quantity of matter contained in any body is measured by comparing its weight with the weight of some particular body. Thus if we take a *cubic inch* of *water* for the body to measure the quantities of matter contained in all other bodies by, and we call the quantity of matter in this cubic inch of water *one*, then the quantity of matter in any other body would properly be said to be *five*, if the weight of that body were five times as great as the weight of the cubic inch of water.]

7. DEF. When we speak of the DENSITIES of different substances, we refer to the closeness with which the matter composing them is packed, as it were, in the substances.

To compare this closeness, let *equal* bulks of two different substances be taken;—suppose the substances to be water and lead. If the bulk of *water* taken weigh *one* pound, it will be found that the piece of lead of equal size will weigh $11\frac{4}{10}$ pounds. There is evidently, therefore, $11\frac{4}{10}$ times as much heavy matter in a piece of lead as there is in an *equal* bulk of water; and this is expressed by saying, that "the DENSITY of lead is to the DENSITY of water, as $11\frac{4}{10}$ to 1."

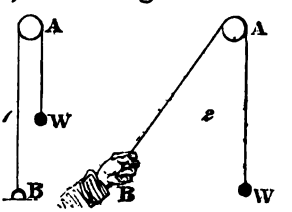
COR. 1. If the density of water be *called* 1,—i. e. if *water* be made the *measure* of density,—then the density of lead will be properly called $11\frac{4}{10}$, or 11·4.

[COR. 2. In the same manner as we have explained how the density of lead is estimated with respect to the density of water, the densities of any other substances, either solid or fluid, may be determined with respect to that of water.]

8. To explain how the precise amount of a statical force may be described;—that is, TO SHEW HOW STATICAL FORCES ARE MEASURED.

The amount of a Statical Force is described by mentioning the number of pounds it would support if it acted in a direction opposite to the force of gravity. Thus if the weight of a body were P pounds, and it were prevented moving towards the earth's surface by a hand placed beneath it, the resistance offered by the hand to the communication of motion, (that is, the *force* exercised by the hand), is clearly P pounds.

9. [FORCES are, in Statics, also called PRESSURES. In whatever direction a force tends to produce motion, its magnitude is, as has already been stated, measured by the weight of the body which would exert the same effect to produce motion downwards, as the force under consideration does in the line in which *it* endeavours to produce motion. And that such a method of measuring Forces is allowable will readily appear from *this* consideration, viz., that the effect produced in consequence of a weight acting may be made to take place in *any* direction we choose; — horizontally, as in the case of a string being attached to an object lying on a table and kept stretched by a heavy body (*W*) that hangs over the side of the table; or vertically upwards, as in fig. 1, by passing the string over a peg *A* and attaching the end to a ring *B*, so that *BA* may be vertical; or in any other direction (fig. 2), by the heavy body pulling the string in a direction *BA* inclined at any angle to that (*AW*) in which it acts itself.



10. (1) The *point* at which a force acts upon a body is called the *point of application of the force.*

(2) The *line* in which a force produces, or tends to produce, motion, is called *the line of the force's action;* and *any* line that is parallel to the line of a force's action is said to be *in the direction* of the force.

[When the *direction of a force's action*, or as it is generally called "*the direction of the force,*" is indicated by a line, we have either the very line given in which the force acts, or some line parallel to it. "*The line of a force's action*"

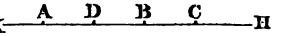
and "*the direction of the force*" must by no means be confounded together. If the former be known, the latter is necessarily known also; but if only the latter be given, the precise line in which the force produces, or tends to produce, motion, is a matter of uncertainty, and all that can be said respecting it is, that the line of action is some line or other parallel to the given line.]

(3) If when two or more forces are applied to a body, or at some point, and no motion is produced, they are said to *counteract*, or to *balance* one another, or to be *in equilibrium*.

11. Forces may be properly represented by lines.

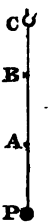
Since lines may be drawn of any length and in any direction from a point, the lines in which forces act, and the ratios that the forces bear to one another, may be represented by drawing lines coinciding with the lines in which the forces act, and bearing to one another the same ratios that the forces themselves bear to one another.

[Among other advantages that attend the indicating in this way the magnitudes and directions of forces, the addition and subtraction of such forces as act at a point in the same straight line is easily effected. Thus if a certain force act at A in the line AH, and we take AB to represent it, and another force, half the size of the former, act at A in the same line, and also tend to move the body from A towards H, then by taking BC equal to the half of AB we get a line AC properly representing the whole pressure at A, both with respect to the *magnitude* of that pressure and *the line in which it acts*. And in like manner, if a force equal to half the original force AB act at A in the line AH, but tends to move the body at A from A towards K, half the pressure of the former force will be counteracted by this new force. Cutting off, therefore, from the line AB a part BD equal to the half of AB, the effective pressure still remaining will be properly represented by AD, with respect to both its magnitude and its line of action.]



12. [NOTE. It will be gathered from the last Article, that a force AB applied at A is not the same thing as a force BA applied at that point; for a force AB would tend to move a body at A in the line KH towards H, but a force BA would tend to move a body at A in the line KH towards K. It is not therefore indifferent whether we say "a force AB" or "a force BA"; since though the two forces represented by AB and BA are exactly the same in *magnitude*, and also act in the same straight line, yet they tend to produce motions that are directly opposite to one another; the force AB tending to move the body at A towards H, and the force BA tending to move the body at A towards K.]

13. [The effect produced by any force is the same at whatever point in its line of action it is applied.



For if a heavy body P be suspended by a string CB, the force necessary to prevent P falling to the earth is the same whether it be applied at A, or B, or C;—the weight of the string being either neglected, or the weight of that portion of it which is supported along with the heavy body P, being counterbalanced, or otherwise accounted for.]

14. [DEF. If a string fastened at one end be pulled by a force applied at the other end, the resistance to motion made by the string at any point in it is called the TENSION of the string at that point.

If the string be supposed to be without weight, it will follow from Art. 13 that the tension at *every* point of it is the same,—namely the force by which it is pulled.]

15. To recapitulate the substance of this Chapter.

(1) MATTER is the substance of which all bodies are composed. It is found, by universal experience, to possess the property of endeavouring to move towards the earth.

(2) The precise amount of the exertion which any particular body makes to move towards the earth is called the WEIGHT of that body.

This WEIGHT of a body is *measured* by determining how much the tendency of the body to move towards the earth is greater than the tendency of some other given body, of a certain size and formed of a certain material, which is taken for a standard, and to which the name of a *grain*, an *ounce*, a *pound*, or a *ton* is given, as the case may be.

(3) THE QUANTITIES OF MATTER contained in different bodies (whether the bodies be great or small, rare like gas or dense like lead), is *proportional*,—(not *equal*,)—to the weights of the bodies.

(4) The DENSITIES of different substances are *proportional*,—(not *equal*,)—to the weights of *equal bulks* of the substances.

(5) I. Whatever moves, or tends to move, matter existing under any form whatsoever, is called FORCE. FORCES that prevent motion taking place,—that is, STATICAL FORCES,—are measured by the number of pounds they would support if they acted vertically upwards.

II. THE LINE OF A FORCE'S ACTION is the line in which the force tends to produce or prevent motion.

III. THE DIRECTION OF A FORCE is indicated either by the line of its action, or by any line parallel to the line of its action.

IV. The magnitudes of forces, and the lines in which they act, may be represented by straight lines properly drawn.

V. If *AB* represent a force acting on a point *A*, and *BA* represent another force acting on the same point, the two forces *AB* and *BA* are equal, and tend to produce motion in the same straight line, but in opposite ways from the point *A*.

VI. To investigate the effects produced by a force we must have given,—1st, the magnitude of the force, which is known if we know how many pounds the force would support; 2nd, the point at which the force is applied; and 3rd, the direction in which it acts; (for knowing the direction of the force and the point it is applied at, the *line* in which the force acts may be determined by drawing a line through the given point parallel to the given direction).

CHAPTER II.

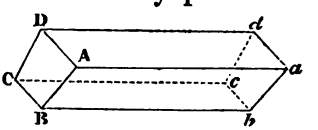
THE LEVER.

16. DEFS. (1) A PLANE, or A PLANE SUPERFICIES, is that superficies, or surface, in which if *any two* points be taken the straight line joining them lies wholly within the superficies. (EUCLID I; Def. 7.)

(2) A SOLID is that which hath length, breadth, and thickness. (EUCLID XI; Def. 1.)

(3) PARALLEL PLANES are such as do not meet one another, however far they may be produced. (EUCLID XI; Def. 8.)

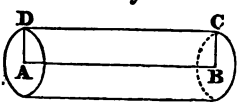
(4) A PRISM is a solid bounded by plane rectilineal figures, two of which, which are parallel to each other, are similar and equal triangles squares or polygons, and all the remaining plane rectilineal figures that bound the prism are rectangles.



[Thus, let *ABCD* and *abcd* be two equal and similar quadrilateral figures, placed with their planes parallel, and let all the figures, (such as *ABba*,) that are formed by joining the equal angles of *ABCD* and *abcd*, be rectangles; then the solid included by these rectangles and by the *ends*, or *bases*, *ABCD* and *abcd*, is called a *prism*.

The *length* of the prism is any one of the edges *Aa*, *Bb*, &c., which lines, being the sides of adjacent rectangles, are all equal to one another.]

(5) A CYLINDER is a solid described by a rectangle (*ABCD*) revolving round one of its sides (*AB*) which remains fixed.



[The side (*AB*) that remains fixed is the *length* of the cylinder, and is called its *axis*.

The surfaces described by the two sides (*AD* and *BC*) of the rectangle which are adjacent to the fixed side, are circles in form, and are parallel to each other; they are called the *ends*, or *bases*, of the cylinder.]

17. DEF. A LEVER is a rigid and inflexible rod, moveable in one plane round a point in the rod called the FULCRUM.

In the following propositions the thickness of the rod is neglected, and the lever is considered to be a geometrical line; it is also supposed to be without weight.

18. The investigation of the properties of the Lever will be made to depend on the following Axioms, which the mind readily admits as true.

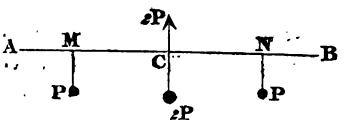
AXIOM I. If two *equal* weights, or forces, act perpendicularly upon a horizontal straight lever, they will balance round a fulcrum placed at the middle point between the points at which they are applied; and the pressure on this fulcrum will be the sum of the weights.

AXIOM II. If any two weights, or forces, acting perpendicularly on a horizontal straight lever, balance each other, the pressure on the fulcrum is equal to the sum of the weights.

Axiom III. The efforts made by two *equal* forces, acting perpendicularly at the extremities of *equal* arms of a lever, to turn the lever round, are equal.

19. PROP. A horizontal prism, or cylinder, of uniform density, will produce the same effect by its weight as if it were collected at its middle point.

Let P and P be two equal weights acting perpendicularly at M and N on the straight horizontal lever AB which is moveable round any fulcrum in it; They produce the same effect as if they were applied at C the middle point between M and N.



At C let a weight $2P$ act perpendicularly downwards, and also let a pressure equal to $2P$ act upwards at C. Since these two forces are equal and opposite they will counteract each other, and the tendency of the lever to turn round the fulcrum will remain unaltered.

Now since P at M and P at N would produce a pressure downwards of $2P$ on a fulcrum at C, (Axiom I.), the pressure upwards exerted by such a fulcrum would be $2P$. The pressure therefore of $2P$, which we have supposed to act upwards at C, will serve instead of such a fulcrum, and will therefore balance P at M and P at N. Wherefore these three forces, (namely, P at M, P at N, and $2P$ acting upwards at C), will have no tendency whatever to turn the lever about, but its tendency to turn about, (which has been shewn to be the same as when the

lever was acted on by no other forces than P at M and Q at N), arises entirely from a force $2P$ applied at C, and acting downwards.

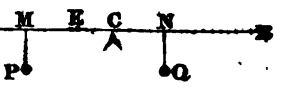
Now supposing* that a horizontal prism (or cylinder) of uniform density, would produce the same effect by its weight, as if the whole matter composing it were collected into a number of small equal weights placed at equal distances along a horizontal straight line of the same length as the prism (or cylinder), then, since it appears from what has been proved above that *every* pair of weights equidistant from the center of the line would produce the same effect as if it were collected at the center, the whole prism therefore, or cylinder, (being made up of such pairs of equal weights), will produce the same effect by its weight as if it were collected at its center. Q. E. D.

Cor. Hence it follows that a horizontal prism, or cylinder, of uniform density will balance on its middle point, and the pressure on a fulcrum placed there will be the weight of the prism, or cylinder.

20. Prop. II. If two weights, acting perpendicularly on a straight lever, on opposite sides of the fulcrum, balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to their sum.

* A uniform prism, or cylinder, may be divided, by means of equidistant planes parallel to its ends, into equal small prisms or cylinders lying on each side of the plane passing through its center. Now if these be supposed to be condensed and placed at equal distances along a straight line equal to the length of the prism, or cylinder, the supposition made in the text will not appear unreasonable.

Let there be an uniform cylinder whose length is AB, and whose weight is $P + Q$. If it be placed horizontally it will balance on a fulcrum C placed at its middle point, (Prop. 1. Cor.), and the pressure on C will be the weight of the cylinder, namely $P + Q$.



Let AB be divided in E so that

$$AE : AB :: P : P + Q;$$

then, since the cylinder is supposed to be of uniform density,

weight of the part AE of the cylinder : weight of the whole

$$:: AE : AB$$

$$:: P : P + Q, \text{ by construction;}$$

but weight of the whole $= P + Q$,

$\therefore$ weight of part $AE = P$;

and $\therefore$ weight of $EB = Q$.

But if the parts AE, EB be collected at their middle points M and N, and act perpendicularly on the lever, they will produce the same effects by their weights as before; Prop. 1. Therefore the weights P and Q, acting perpendicularly at M and N, will balance about C, and the pressure on C will be $P + Q$.

Now

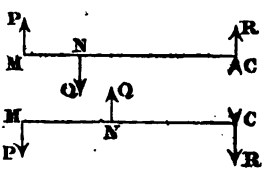
$$\frac{P}{Q} = \frac{AE}{EB} = \frac{AB - BE}{AB - AE} = \frac{2BC - 2BN}{2AC - 2AM} = \frac{BC - BN}{AC - AM} = \frac{CN}{CM},$$

$$\text{or } P : Q :: CN : CM.$$

Wherefore, "If two weights, &c." Q. E. D.

21. PROP. III. If two forces, acting perpendicularly on a straight lever in opposite directions and on the same side of the fulcrum, balance each other, (1) they are inversely as their distances from the fulcrum, and (2) the pressure on the fulcrum is equal to this difference.

Let the forces P and Q, acting perpendicularly at M and N on the straight lever MC in opposite directions and on the same side of the fulcrum C, balance each other; Then $P : Q :: CN : CM$, and, (Q being the force which is the nearer to the fulcrum), the pressure on the fulcrum $= Q - P$.



Let the fulcrum at C be removed, and the place of its resistance (R) supplied by a force acting perpendicularly to the lever in the direction of P's action.

Since P and R are counterbalanced by Q, they must produce a pressure at N equal and opposite to Q. Let Q be removed, and its place supplied by a fulcrum on the contrary side of the lever to that on which Q acted, sustaining the pressure (namely Q) which P and R produce. The equilibrium therefore still existing round N, we have by Prop. II.,

$$P : R :: CN : NM$$

$$\therefore \quad P : P + R :: CN : CN + NM :: CN : CM.$$

But Q, the pressure on the fulcrum at N, is (by Prop. II.) $= P + R$.

$$\therefore \quad P : Q :: CN : CM.$$

Also, $R = Q - P$.

Wherefore, "If two forces, &c." Q. E. D.

22. [From the last two Propositions it appears, that if a straight lever be at rest, which is acted on perpendicularly by two weights or other forces P and Q respectively applied at the distances CM and CN from the fulcrum C, then, whether P and Q act on the same or on different sides of the fulcrum, the proportion

$$P : Q :: CN : CM$$

is always true.

When the Lever is used to balance a given weight or force Q by the application of another force P, Q is usually called "*the Weight*", and P "*the Power*".

If CM, the perpendicular distance from the fulcrum at which the Power acts, be greater than CN the distance at which the Weight acts, the Power required to balance the Weight is less than it. In this case "*force*" is said "*to be gained*" by the application of the lever. But if CM be less than CN, the Power required to balance the Weight is greater than it, and "*force*" is then said "*to be lost*".]

23. PROP. IV. To explain the different kinds of levers.

LEVERS are divided into three classes.

(1) Where the Power and the Weight act on *opposite* sides of the fulcrum.

(2) Where the Power and the Weight act on the *same* side of the fulcrum, but the perpendicular distance at which the Power acts is *greater* than that at which the Weight acts.

(8) Where the Power and the Weight act on the *same* side of the fulcrum, but the distance at which the Power acts is *less* than that at which the Weight acts.

Of the FIRST class the poker, when used to raise the coals, is an instance; the bar of the grate on which the poker rests being the Fulcrum, the force exerted by the hand the Power, and the resistance of the coals the Weight. In the common balance, the Power and the Weight are equal forces perpendicularly applied at the ends of equal arms. In the steelyard, the Power, and the Weight are perpendicularly applied at the ends of unequal arms. Pincers, scissars, and snuffers, are double levers of this kind, the rivet being the fulcrum in each.

Since *CM* may be either greater or less than *CN*, the Power in levers of this class may be either less or greater than the Weight, and consequently "Force" may be either "gained" or "lost" by using them.

Of the SECOND class, a cutting blade, such as those used by coopers, moveable round one end by means of a handle fixed at the other, is an example. An oar is also such a lever, the Fulcrum being the extremity of the blade which remains fixed, or nearly so, during the stroke, the muscular strength and weight of the rower being the Power, and the Weight being the resistance of the water to the motion of the boat, which is counteracted and overcome at the rowlocks. A pair of nutcrackers is a double lever of the second class.

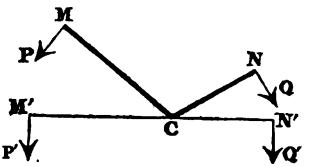
Here, since *CM* is greater than *CN*, the Power is always less than the Weight, or "Force is gained" by using levers of the *second* class.

An example of the THIRD class is the board which the turner (or knifegrinder) presses with his foot to put the wheel of his lathe in motion; the Fulcrum being the end of the board which rests on the ground, the Power being the pressure of the foot, and the Weight being the pressure produced at the crank on the axletree of the wheel. Fire tongs, or sugar tongs, are double levers of this kind; the Weight being in either case the resistance of the substance grasped. The limbs of animals are also such levers. Thus, if a weight be held in the hand and the arm be raised round the elbow as a Fulcrum, the weight is supported by muscles fastened at one extremity to the upper arm, and again attached to the forearm, after passing through a kind of loop at the elbow.

Here, since CM is less than CN, the Power is greater than the Weight, or "Force is lost" by making use of levers of the *third* class.

24. PROP. V. If two forces acting perpendicularly at the extremities of the (straight) arms of *any* lever, balance each other, they are inversely as the arms.

Let the forces P and Q, acting perpendicularly at the extremities of the straight arms CM and CN of any lever whose fulcrum is C, balance each other; They are inversely as the arms.



Draw the horizontal line $P'Q'$ through C, and make $CM' = CM$ and $CN' = CN$. Let a force P', equal to P, act perpendicularly at M', and a force Q', equal to Q, act at N'.

Now since $CM' = CM$ and $P' = P$, P' will produce the same effect as P on the lever; Axiom III. Art. 18;

But, by supposition, P balances Q,

$\therefore$ P' would balance Q.

And since $CN' = CN$, and $Q' = Q$, Q' will produce the same effect on the lever as Q; Ax. III.

But Q would balance P',

$\therefore$ Q' would balance P';

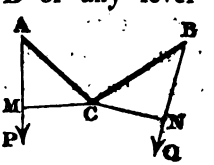
wherefore, by Prop. II. $P' : Q' :: CN' : CM'$,

and $\therefore$ $P : Q :: CN : CM$.

Wherefore, "If two forces, &c." Q. E. D.

25. PROP. VI. If two forces acting at any angles on the arms of any lever, balance each other, they are inversely as the perpendiculars drawn from the fulcrum on the directions, (i. e. the lines), in which the forces act.

Let P and Q be two forces which acting at any angles on the arms CA and CB of any lever ACB, balance each other about the fulcrum C; They are inversely as the perpendiculars CM and CN drawn from the fulcrum on the lines in which they act.



Since a force produces the same effect at whatever point in its line of action it is applied (Art. 15), we may suppose the force P applied at M; and that it *may* be so applied let a rod (CMA)

without weight be fastened to CA. In like manner Q may be supposed to be applied at N perpendicularly to the part CN of the rod CNB which is added to CB.

And since P acting at M perpendicularly to CM balances Q acting at N perpendicularly to CN, we have by Prop. v.,

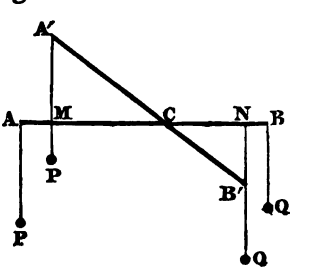
$$P : Q :: CN : CM;$$

and therefore when P and Q acting at A and B in the lines AMP, BNQ balance, they are as CN : CM.

Wherefore, "If two forces, &c." Q. E. D.

26. PROP. VII. If two weights balance one another on a straight lever when horizontal, they will balance each other in *every* position of the lever.

Let P and Q be two weights that balance each other round the fulcrum C on the lever ACB when it is horizontal; They will balance each other on the lever when it comes into any other position, as $A'CB'$.



Produce QB' to cut AB in N.

Since weights act perpendicularly to the horizon, $A'P$ and $NB'Q$ are each perpendicular to the horizontal line ACB;

$\therefore$ angle CMA = right angle = angle CNB',

and angle $A'CM$ = opposite angle $B'CN$; Euc. I. 15.

$\therefore$ also angle $CA'M$ = angle $CB'N$, and the triangles are equ angular.

Therefore by Euc. VI. 4,

$$CN : CB' :: CM : CA';$$

$$\therefore CN : CM :: CB' : CA' :: CB : CA.$$

But $CB : CA :: P : Q$, $\because$ P and Q balance on ACB;

$$\therefore CN : CM :: P : Q;$$

But CM and CN are perpendiculars from C to the lines in which P and Q act when they are hung at A' and B'; therefore, by Prop. VI., P and Q will balance on $A'CB'$.

Wherefore, "If two weights, &c." Q. E. D.

CHAPTER III.

THE COMPOSITION AND RESOLUTION OF FORCES.

27. DEFINITION of COMPONENT, and RESULTANT Forces.

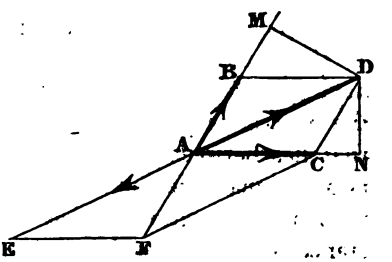
A body that is acted on by two forces applied at the same instant to the same point, instead of having a tendency to move in either of the lines the forces act in, has a tendency to move in a line that lies between them. Whence it appears that the two original pressures produce a third pressure by their mutual action, which is called, from the circumstance of its "resulting" out of the action of the original pressures, their "Resultant" with respect to them, while they are called, with respect to it, its "Components."

The Resultant (R) which produces the same effect as the compound action of the original forces P and Q that are applied at the same point, is said to be "compounded" of P and Q. This Resultant (R) also, if conceived to be the sole original force, may be supposed to be "resolved" into the two forces P and Q; since those two forces, acting in the manner described, (namely, at the same point, and at the same instant), produce exactly the same effect on the body as the single force R does.

[An instance of the "Composition of Forces" is a boat moored in a stream by ropes attached to the boat and to either bank; the Component forces here being the tensions of the ropes, and their Resultant being the force by which the pressure of the stream on the boat is counteracted.]

28. PROP. VIII. If the adjacent sides of a parallelogram represent the component forces, [that act at a point at the same instant] in direction [i. e. in their lines of action] and magnitude, the diagonal will represent the resultant force (1) in direction [line of action], and (2) in magnitude.

Let *AB* and *AC* represent the two component forces that act at *A*, in direction and in magnitude. Complete the parallelogram *BC*, and draw the diagonal *AD*; Then *AD* will represent the resultant of *AB* and *AC* in (1) direction, and also in (2) magnitude.



From *D* draw *DM* and *DN* perpendicular to *AB* and *AC*, produced if necessary.

(1) Then in the triangles *DBM*, *DCN*,

angle *DMB* = right angle = angle *DNC*,

And angle *DBM* = angle *BAC* (since *BD*, *AC* are parallel and *MBA* cuts them) = angle *DCN*;

∴ the third angle, *BDM*, of the one triangle = the third angle, *CDN*, of the other; and the triangles are equiangular.

∴ *CD* : *DN* :: *BD* : *DM*

and ∴ alternately, *CD* : *BD* :: *DN* : *DM*.

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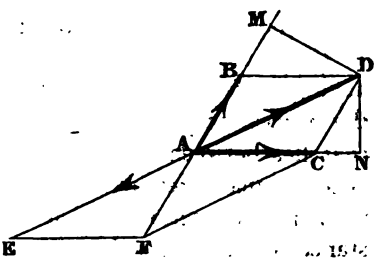
Let AB and AC represent the two component forces that act at A, in direction and in magnitude. Complete the parallelogram BC, and draw the diagonal AD; Then AD will represent the resultant of AB and AC in (1) direction, and also in (2) magnitude.



From D draw DM and DN perpendicular to AB and AC, produced if necessary.

(1) Then in the triangles DBM, DCN,

angle DMB = right angle = angle DNC,

And angle DBM = angle BAC (since BD, AC are parallel and MBA cuts them) = angle DCN;

$\therefore$ the third angle, BDM, of the one triangle = the third angle, CDN, of the other; and the triangles are equiangular.

$$\therefore CD : DN :: BD : DM$$

and $\therefore$ alternately, $CD : BD :: DN : DM$.

Now considering AD as a lever, fulcrum D, (see fig. on p. 25), that is acted on by the forces AB, AC applied at A, since we have

Force in the line AB : force in line AC :: AB : AC

:: CD : BD, since BC is a parallelogram,

:: DN : DM,

$\therefore$ the two forces acting on the lever AD are inversely as the perpendiculars from the fulcrum on their lines of action, and therefore the lever will be kept at rest about D by them; wherefore the lever will also be kept at rest by the Resultant of those forces, which produces the same effect as they do when they act at the same point and at the same instant.

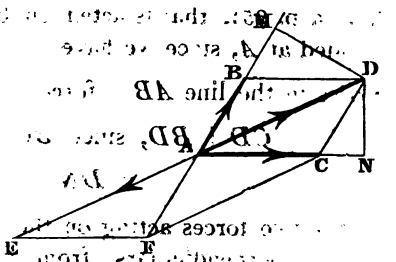
This Resultant therefore *must act in the line* AD, for it keeps the lever at rest, which it could not do were it to act at A and make any angle with the lever AD on either side of it. Q. E. D.

(2) Again: Having shewn that the resultant of AB and AC acts in the line of the diagonal, next to prove that the diagonal represents it in *magnitude* as well as in direction.

Produce DA, and suppose a force AE to be taken equal and opposite to the Resultant of AB and AC. The effects of AB and AC will now be counteracted by AE, and the point A, which is acted on by the three forces AB, AC, and AE, will remain at rest. Whatever be the effect therefore that is produced by the joint action of AE and AC, it is counteracted by AB; that is, AB must be equal and opposite to the Resultant of AE and AC.

Complete the parallelogram EC, and draw the diagonal AF. By the first part of the Prop. AF

is the line which the resultant of *AE* and *AC* acts in; therefore the force *AB* must be equal and opposite to that resultant, because it destroys its effect; and therefore *AF* must be in the same straight line with *AB*.

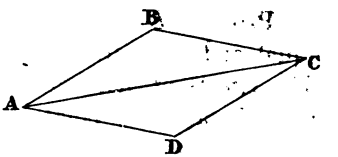


Wherefore *EC* and *FD* are parallelograms; and since $AE = FC = AD$, the Resultant of *AB* and *AC* (which is equal and opposite to *AE*) will be properly represented in *magnitude* by *AD* the diagonal of the parallelogram of which *AB* and *AC* are the sides.

Wherefore, "If the adjacent sides, &c." Q. E. D.

29. PROP. IX. If three forces represented in magnitude and direction by the three sides of a triangle [when taken in order], act on a point, they will keep it at rest.

Let the sides *AB*, *BC*, *CA*, taken in order*, of the triangle *ABC*, represent in magnitude and direction (see Art. 15), three forces that act on the point *A*; They will keep *A* at rest.



Complete the parallelogram *BD*.

* By the expression "taken in order" is meant, that if *ABC* be the triangle and *AB* be one of the forces, *BC* (and not *CB*) is the next, and *CA* (not *AC*) is the third; so that in describing the forces we proceed regularly round the triangle, from *A* to *B*, from *B* to *C*, and from *C* to *A* again.

Then AD is parallel and equal to BC, and will therefore properly represent in magnitude and in line of action (see Art. 15) the force which acts at the point A in the direction BC.

Now the forces AB and AD acting at A will produce a resultant AC. Prop. VIII.

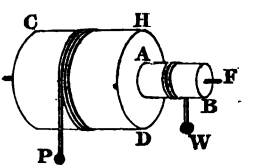
If therefore a force CA act at A, the force AC will be counteracted, and the point A will remain at rest; Wherefore, if three forces, represented in magnitude and direction by AB, AD, CA,—(or, which is the same thing, if they be represented by the three sides AB, BC, CA taken in order of the triangle ABC)—act on a point A, they will keep it at rest. Q. E. D.

CHAPTER IV.

MECHANICAL POWERS.

30. THE MECHANICAL POWERS are certain instruments by which a lesser weight, called the *Power*, may be made to balance a greater, called the *Weight*. We are now going to investigate the properties of three of them, which bear the names of the *Wheel and Axle*, the *Pulley*, and the *Inclined Plane*. They will be supposed to be rigid bodies and without weight.*

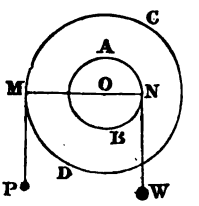
31. THE WHEEL AND AXLE consists of a cylinder (or *Axle*) *AB* that terminates in another cylinder (or *Wheel*) *CD* of greater base. The two cylinders have a common axis *EF* which is supported in a horizontal position. The (*Power*) *P* is a heavy body that hangs freely by a string coiled round the *Wheel*, and the *Weight* (*W*) is another body hanging by a string that is coiled round the *Axle* and tending to turn the machine round in an opposite direction.



PROP. X. There is an equilibrium on the Wheel and Axle when the Power is to the Weight as the radius of the Axle is to the radius of the Wheel.

* The other Mechanical Powers are the Lever, the Toothed Wheel, the Wedge, and the Screw.

Suppose the strings that the Power and the Weight hang by to be in the planes of the circles *HD* and *AG*, which are those ends of the two cylinders that lie in the same plane. Since the cylinders have a common axis, their circular ends have a common center; let *O* be it, and join *O* with *M* and *N* the points in which the strings leave (and therefore touch) the circumferences of the circles. *OM* and *ON* are therefore perpendicular to *MP* and *NW* the lines in which the Power and the Weight act.



Now since *P* and *W* acting perpendicularly on the arms *OM* and *ON* balance each other, we have, supposing *MON* a lever moveable round *O* as fulcrum,

P : *W* :: *ON* : *OM* :: radius of axle : radius of wheel.

When therefore this ratio of the Power to the Weight is fulfilled on the Wheel and Axle, there will be equilibrium. Q. E. D.

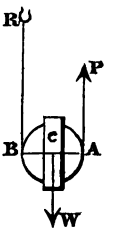
[Cor. Since *MO* and *NO* are both perpendicular to vertical lines and also pass through the same point *O*, *MON* is a horizontal straight line.]

32. Def. The Pulley consists of *a grooved wheel* moveable round an axis whose ends are fixed in a frame called *the Block*.

When the Pulley is used as a Mechanical Power, a string, which is fastened at one end, is passed round part of the circumference of the wheel, and the Power is applied at the other end of the string. The Weight is attached to the Block.

PROP. XI. In the single moveable pulley where the strings are parallel, there is an equilibrium when the Power is to the Weight as 1 to 2.

Let PA, which touches the pulley at A, be the line in which the Power (P) acts. Join A with C the center of the pulley. Produce AC to B. Draw BR touching the circle; it is perpendicular to AB and parallel to AP; this therefore will be the other part of the string. Let W, the weight, act at C by means of the block, in the line CW.



The whole being at rest, the string $RBAP$ must be equally stretched throughout, or motion would ensue. The force upwards therefore exerted by BR is P, for it is the same as that which keeps AP stretched.

Now considering ACB as a straight lever kept at rest round C by forces P and P acting perpendicularly at A and B, the pressure upwards on C will be $2P$ (Prop. II); wherefore the weight (W) which acts downwards at C and is supported by this pressure, is $= 2P$;

$$\text{or} \quad P : W :: 1 : 2.$$

Wherefore, "In the single moveable pulley, &c." Q. E. D.

[COR. The pressure produced at C by the forces P and P acting at A and B is perpendicular to AB, or parallel to AP and BR; wherefore the line of action of the Weight is perpendicular to AB, or

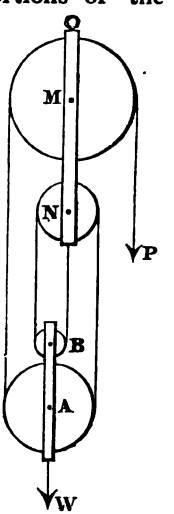
parallel to AP and BR. If therefore "the Weight" be a heavy body hanging freely from C and acted on by the force of gravity, since in a case of equilibrium the strings are parallel to the line in which the weight acts, AP and BR are vertical.]

33. THE FIRST SYSTEM OF PULLEYS;—where the same string passes round any number of pulleys.

PROP. XII. In a system in which the same string passes round any number of pulleys, and the parts of it between the pulleys are parallel, there is an equilibrium when Power (P): Weight (W) :: 1 : number of strings at the lower block.

[In this Proposition, as in Prop. XI., the Power and the Weight are supposed to act in a direction parallel to that of the strings.]

Since there is equilibrium, the portions of the string that lie on either side of any pulley must be pulled by equal forces; and since the outside portion, to which the power is applied, is acted on by a force P, each of the strings is therefore pulled by a force P.



Wherefore the part of the weight sustained at A is (by Prop. XI) $2P$, and the same is true with respect to the pulley B, and so on for every other pulley at the lower block. If therefore there be n pulleys at the lower block, W, the whole weight supported, will be $n \times 2P$; and we have,

$$W = 2nP; \qquad \therefore \frac{P}{W} = \frac{1}{2n}$$

or $P : W :: 1 : 2n :: 1$: number of strings at the lower block.

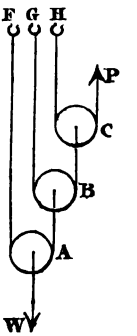
Wherefore, "In a system, &c." Q. E. D.

34. THE SECOND SYSTEM OF PULLEYS;—in which each pulley hangs by a separate string.

PROP. XIII. In a system where each pulley hangs by a separate string and the strings are parallel, there is an equilibrium when $P : W :: 1 : 2$ raised to that power whose index is the number of moveable pulleys.

[The weight W is supposed to act in a line parallel to the strings.]

In the figure, C being a moveable pulley with parallel strings, there is equilibrium if the pressure downwards at $C = 2P$.



$\therefore$ Tension of $BC = 2P$

$\therefore$ Weight supported at moveable pulley B

$= 2 \times (2P)$

$= 2^2 P$

= tension of AB;

So Weight supported at moveable pulley A

$= 2 \times (2^2 P)$

$= 2^3 P$.

Wherefore, if W, "the Weight," be $= 2^3 P$, there is equilibrium on the system here represented.

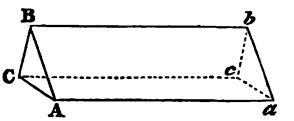
And in the same manner, if n be the number of moveable pulleys, it would appear by the same mode of reasoning that there will be equilibrium when

$$W = 2^n P, \quad \text{or} \quad \frac{P}{W} = \frac{1}{2^n}.$$

$$\text{or} \quad P : W :: 1 : 2^n.$$

Wherefore "In a system where each pulley hangs by a separate string, &c." Q. E. D.

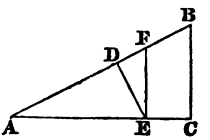
35. DEF. If a prism whose ends are similar and equal right-angled triangles, (C and c being the right angles), be placed so that either of the sides of the prism adjacent to the right angles (as $ACca$) is horizontal, the slant side $ABba$ is called an *Inclined Plane.*



When the Inclined Plane is used as a Mechanical Power, both the Power and the Weight act in a plane parallel to either end of the prism, and the plane is represented by a triangle such as ABC, of which the side AB is called the *Length* of the plane, AC its *Base*, and CB its *Height*. And the plane, being supposed to be perfectly rigid and inflexible, will therefore be capable of counteracting and entirely destroying any force that acts upon it in a line perpendicular to it.

PROP. XIV. The weight (W) being on an Inclined Plane and the force (P) acting parallel to the plane, there is an equilibrium when $P : W ::$ the height of the plane : its length.

Let AB be the length of the plane; AC its horizontal base; and BC, perpendicular to AC, its height. Let the Weight (W) act vertically at D, and the Power (P) act in the line DF which is parallel to the plane; Then if $P : W :: BC : AC$, there is equilibrium.



Draw DE at right angles to AB, meeting the base in E; and EF vertical, or at right angles to AE.

Then in the triangles EFD and ABC,

angle DFE = angle ABC, $\because$ FE, BC are parallel, and AFB cuts them;

and angle FDE = right angle = angle BCA,

therefore angle DEF = angle CAB, and the triangles are equiangular.

Now, by hypothesis,

$P : W :: BC : AB$

$:: DF : FE$, by equiangular triangles.

Now the same effect that is produced on D by the two forces that are represented in magnitude and in their lines of action by DF and FE, will be produced by their resultant. But their resultant (DE), being perpendicular to the plane AB, will be entirely counteracted and destroyed by that plane, and no motion would be communicated to D. The body at D, therefore, will remain at rest when acted on by the two forces P and W which bear the ratio to each other of $BC : AB$.

Wherefore, "The Weight, &c." Q. E. D.

36. DEF. By the VELOCITY of a body in motion is meant the degree of quickness with which the body is moving.

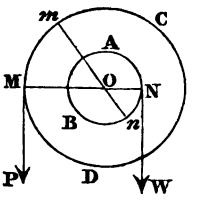
[This "degree of quickness" is described, or *measured*, by saying how long the line is that the body moves through in some *given* portion of time. Thus a clear notion would be conveyed of the Velocity of a coach, if it were said to be *nine miles* in *an hour*,—the *space* moved over by the coach being *nine miles*, and *an hour* being the *portion of time* during which the motion takes place. If only the space that is described were mentioned, and nothing were said about the time taken up in describing that space, —or if the time alone were given,—it is evident that no idea at all could be formed of *the degree of quickness*,—that is, of the *Velocity*,—of the coach's motion.

The Velocity of a body is measured, in mathematical investigations, by the number of *feet* passed over by the body in a *second* of time.

Since the quicker a body moves the more space it will pass over in a *given* time, it will follow from the observations just made, that *The Velocities of two bodies that move during any* (THE SAME) *time, are as the spaces the bodies respectively describe in that time.*]

37. PROP. XV. Assuming that the arcs which subtend equal angles at the centers of two circles are as the radii of the circles, to shew that if P and W balance each other on the Wheel and Axle, and the whole be put in motion, $P : W :: W$'s velocity : P's velocity.

Suppose P and W to act at the circumferences of the Wheel and the Axle in the same plane perpendicularly to the radii OM and ON (as in Prop. x.); and let the whole be put into motion round the axis so that mOn becomes the horizontal diameter. P will now act at m at right angles to Om, and W will act at n at right angles to On; and the velocities of P and W will be as Mm to Nn, since the former of these arcs is the length through which P will have descended in the same time that W has ascended through Nn.



Therefore vel. of P : vel. of W :: Mn : Nn

:: OM : ON

[by the assumption made in the enunciation],

:: W : P,

by Prop. x; because P and W originally balanced each other.

Wherefore, "Assuming, &c." Q. E. D.

38. PROP. XVI. To shew that if P and W balance each other on the Machines described in Propositions XI, XII, XIII, and XIV, and the whole be put in motion, P : W :: W's velocity in the direction of gravity : P's velocity [in the direction in which it acts.]

(1) Let C the center of the moveable pulley (see Fig. Prop. XI), be raised through any height, as an inch, W will be raised through an inch, and each of the strings RB and AP will have been shortened an inch, so that if P continue to keep the string tight, it will have moved through two inches in the time that W has been raised one inch.

Now $P : W :: 1 : 2$ by Prop. XI,

:: 1 inch : 2 inches

:: W's velocity : P's velocity; by Art. 36. Q. E. D.

(2) In the system when the same string passes round all the pulleys, and the parts of it between the pulleys are parallel, as in Prop. XII, (see Fig.), if the lower block be raised an inch, each of the strings between the pulleys will be shortened an inch, and (there being $2n$ of such strings) P must have moved through $2n$ inches in the time that W moved through one inch, in order to have kept the string tight.

And $P : W :: 1 : 2n$

by Prop. XII.

:: 1 inch : $2n$ inches,

:: W's velocity : P's velocity; by Art. 36. Q. E. D.

(3) In the system where each pulley hangs by a separate string and the strings are parallel (Prop. XIII), if W be raised through an inch, then if P have moved through such a space so as to keep the strings tight, A will have been raised through one inch; B through two inches (by the first case proved in this Proposition); and B having been raised through two inches, C (by the first case) will have moved through 2×2, or 2^2, inches; the next moveable pulley (the

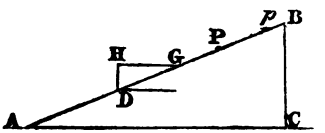
fourth) would have been raised through 2×2^2 (or 2^3) inches; and by the same kind of reasoning, if n were the number of moveable pulleys, the highest of them would have moved through 2^{n-1} inches. The end therefore of the string by which P acts would have moved through 2^n inches; and

$P : W :: 1 : 2^n$, by Prop. XIII.;

$:: 1$ inch $: 2^n$ inches

:: W's velocity : P's velocity. Q. E. D.

(4) Let the weight (W) be kept at rest at D on the inclined plane AB by the power (P) which acts parallel to the plane by means of a string DP. Suppose P moves through the space Pp, raising W through an equal space DG. Through G draw GH horizontal, and DH through D vertical. Then by being moved through DG, W has been raised through a vertical height DH,—that being the vertical line cutting the horizontal lines passing through the two positions of W. In the time therefore that P moves through a space equal to DG, the vertical height that W moves through is DH, and therefore these two lines are to each other as the velocity of P in the direction of its action is to the velocity of W in the direction of gravity.



Now, in the triangles GDH and ABC,

since GH is parallel to AC, and AG meets them,

$\therefore$ angle DGH = alt. angle BAC;

and since DH, being vertical, is parallel to BC, and DB meets them,

$\therefore$ angle GDH = alt. angle ABC;

$\therefore$ also angle DHG = angle BCA, and the triangles are equiangular.

And

$P : W :: BC : AB$; by Prop. XIV.;

$:: DH : DG$, by equiangular triangles.

$:: DH : Pp$,

:: W's vel. in direction of gravity : P's vel. in the direction of its action.

Wherefore, "If P and W balance, &c." Q. E. D.

CHAPTER V.

THE CENTER OF GRAVITY.

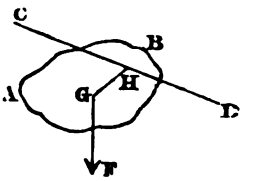
39. DEF. THE CENTER OF GRAVITY of a body, or of a system of any number of bodies connected together, is that point upon which the body (or the system) will balance when placed in any position whatever;—the point itself being maintained at rest, and the body (or the system) being acted on by no other force than the pressure arising from the weight of the matter composing it.

[The Center of Gravity is (as it were) that fulcrum, round which the body, when placed in any position, has no kind of tendency whatever to turn, although the body be *capable* of being moved in any plane, and in any direction, about that fulcrum.]

40. PROP. XVII. If a body balance itself upon a line in all positions, the center of gravity of the body is in that line.*

* That is to say; If there be a line round which a body can be made to revolve, such that if when the line is put into *any* position, the body (after being made to revolve round the line into *any* position) remains at rest without having any tendency to move, the center of gravity of the body lies in that line.

If possible, let G, the center of gravity of the body AB, *not* lie in the line CD on which the body balances in all positions; and let the body and line be so placed, that G and CD may be in the same horizontal plane. From G draw GH perpendicular to CD, and GF vertical.



Now since (by Def. Art. 39,) if G be supported the body remains at rest when placed in any position, the vertical pressure which arises from the weight of the matter composing the body must always pass through G, or else that pressure could not be rendered inoperative (as it is supposed to be) by G being maintained at rest. This pressure therefore, in the position of the body represented in the figure, will act perpendicularly at an arm GH, and will have a tendency to turn the body round the point H, which, since it is a point in a line CD fixed in position, may be considered to be a fixed fulcrum. But, by hypothesis, when CD is supported the body balances in *all* positions upon it. Wherefore G does not lie out of the line CD, i. e. it lies in it. Q. E. D.

42. PROP. XVIII. To find the center of gravity of two heavy points*, and to shew that the pressure at the center of gravity is equal to the sum of the weights in all positions.

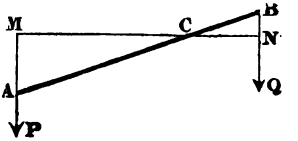
* The "heavy points" spoken of in this Proposition and the next, are exceedingly small material bodies, and not geometrical points, which (being defined by Euclid to be "without parts") are without length, or breadth, or thickness, and therefore can be of no weight, since they can contain no Matter.

Let P and Q be the weights of two heavy points A and B. Join AB, and take

$AC : AB :: Q : P+Q$;

and $\therefore AC : AB-AC$, or CB,

$:: Q : (P+Q)-Q$, or P.



Let ACB be in any position. Through C draw MCN horizontal; and through A and B draw the vertical lines PAM and QNB,—these last are the lines in which the weights P and Q act.

Then the angles at M and N being right angles, and the angle ACM being equal to the opposite angle BCN, the triangles ACM and BCN are equiangular.

Now supposing P to act on the horizontal lever MCN at M at right angles to CN, (Art. 15), and Q at N at right angles to CN, since

$P : Q :: CB : CA$

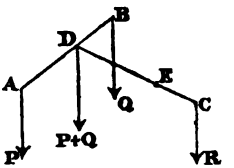
$:: CN : CM$, by equiangular triangles,

therefore P and Q will balance, and the pressure on C is $P+Q$, acting vertically. Props. II. and VII.

And this, being true for *any* position, is true for *all* positions of A and B. Wherefore, by definition of Center of Gravity, C is the Center of Gravity of A and B, and the pressure at C is $P+Q$, acting vertically downwards.

43. PROP. XIX. To find the center of gravity of any number of heavy points; and to shew that the pressure on the center of gravity is equal to the sum of the weights in all positions.

Let A, B, C, three heavy points whose weights are P, Q, R, be connected together and placed in any position.



Join AB, and take D a point in AB such that

$$AD : AB :: Q : P + Q;$$

$$\therefore AD : AB - AD, \text{ or } DB, :: Q : \{P+Q\} - Q, \text{ or } P;$$

therefore D is the center of gravity of A and B, and the pressure produced by P and Q in all positions of the system, is a pressure $P + Q$ acting vertically at D. Prop. XVIII.

Join DC, and take in DC a point E such that

$$DE : DC :: R : P + Q + R,$$

$$\therefore DE : EC :: R : P + Q;$$

therefore E is the center of gravity of the weights $P + Q$ acting at D and R acting at C; and if E be supported, those weights are supported in any position of the system. Since therefore the system will balance itself in all positions on E, that point is its Center of Gravity;—and the Pressure on E is $P + Q + R$.

The construction here applied to a system of three bodies is applicable to a system of any number of bodies.

Wherefore, "The center of gravity of any number of heavy points may always be found, and the pressure on the center of gravity is equal to the sum of the weights." Q. E. D.

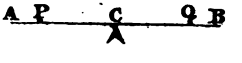
44. [By the definition given in Art. 39, of "the Center of Gravity" of a body, it will be understood that to have a center of gravity a body must have *Weight.* Now in the next two Propositions we are required to find the centers of gravity of a *line*, and of a *plane;* the former of which is defined by Euclid to have *length* merely, without either *breadth* or *thickness*, and the latter, though possessing *length* and *breadth*, is defined to be without *thickness.* A geometrical line, or plane, therefore, *can* have no weight, since there can be no weight where *matter* does not exist, and *matter* existing under any form is of *three* dimensions,—or has *length*, and *breadth*, and *thickness.* The line therefore and the plane, of which we have to find the centers of gravity, are not the line and plane of geometry.

But the line of which the center of gravity is determined in the next Proposition, is supposed to be formed of very small equal heavy bodies placed at the same distance one from the other along the whole length of the line. And the plane triangle referred to in the next Proposition but one, is supposed to be made up of such lines arranged parallel to any one of the sides of the triangle, and at equal distances one from the other.]

45. PROP. XX. To find the center of gravity of a straight line.

Let *AB* be a straight line composed of small equal heavy bodies ranged at equal distances one from the other from end to end of it.

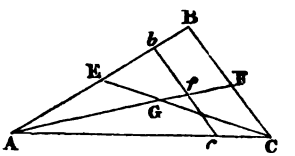
Bisect *AB* in *C*, and let *P* and *Q* be two of the small heavy bodies which are equally distant from *C*. Then



P and *Q* will balance in every position on *C*. And since the same is true of all such pairs of heavy bodies equidistant from *C*, the whole line will balance on *C* in every position, and therefore *C* is the center of gravity of the line.

46. PROP. XXI. To find the center of gravity of a triangle.

Let *ABC* be a triangle formed of lines ranged, at equal distances, parallel to any one of the sides, the lines themselves being made up of small equal heavy bodies placed at equal distances. Let *bfc*, parallel to *BFC*, be one of these lines.



Bisect *AB* in *E* and *BC* in *F*; join *AF*, *CE* by lines intersecting in *G*;—*G* is the center of gravity of the triangle.

For *Af* : *fb* :: *AF* : *FB*, (from the equiangular triangles, *ABF*, *Abf*),

:: *AF* : *FC*, ∵ *BF* = *FC*.

:: *Af* : *fc*, (from the equiangular triangles, *AFC*, *Afc*).

And since the first and third terms of this proportion are the same, *fb* = *fc*, and therefore the straight line *bc* would balance in any position on *f*. Prop. xx.

In the same manner all the other lines parallel to *BC* might be shewn to balance in any position on the points that *AF* cuts them in, therefore the

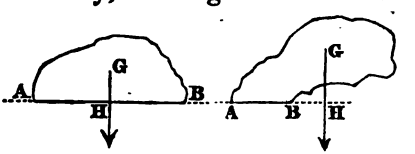
whole triangle would balance on AF in any position, and therefore the center of gravity of the triangle is in AF.

Similarly it may be shewn that the center of gravity of the triangle is in the line CE.

Wherefore G, the intersection of AF and CE, is the center of gravity of the triangle ABC.

47. PROP. XXII. **When a body is placed on a horizontal base, it will stand or fall, according as the vertical line drawn from its center of gravity falls within or without the base.**

Let GH be a vertical line drawn through the center of gravity of the body, meeting the horizontal plane on which the body is placed in H; and let ABH be a horizontal line drawn through H and terminated at A and B, points in the boundary of the base of the body.



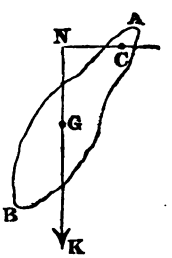
(1) Let H fall *between* A and B. Then AB being considered as a horizontal lever acted on at H by the weight of the body, no motion can take place round A, because the tendency of the weight would be to draw the lever *downwards* round A, a motion which the resistance of the plane will prevent taking place. For the same reason no motion can take place round B, and therefore the body will remain at rest, if the vertical drawn through the center of gravity fall within the base.

(2) If H fall *without* AB, as in the second figure, no motion of the lever ABH can take place round A as fulcrum, because the effect of the weight acting at H would be to draw AH *downwards*, a motion which the plane prevents. But if B be considered as the fulcrum, the effect of the weight at H would be to make the line AB to move *upwards* from the plane round B, and as there is no force to prevent this motion taking place, the body will fall over at B.

Wherefore, "When a body, &c." Q. E. D.

48. PROP. 23. When a body is suspended from a point [round which it can swing freely], it will rest with its center of gravity in the vertical line passing through the point of suspension.

Let AB be the body in any position, C the point from which it is suspended, G its center of gravity, NGK a vertical line drawn through G, CN a perpendicular from C on NGK.



Then since the weight of the body acts in the vertical through G its center of gravity, it may be supposed to be applied at N; and if CN be considered as a lever moveable round C as fulcrum, since there is no force to counteract W acting perpendicularly at N on CN, motion must ensue.

But if NG pass through C, CN vanishes, the weight is sustained by the immoveable fulcrum C, and the body is at rest.

Wherefore "When a body, &c." Q. E. D.

HYDROSTATICS.

THE few explanatory articles and paragraphs that are enclosed in brackets form no part of the Course required to be read by the Candidate for the degree of B.A. *Those in smaller type are illustrative of the Definitions and Propositions which they immediately follow. The whole of the extra matter so introduced forms but a very small, and a very easy, portion of the book; and they who come to this subject for the first time will do well not to omit it.*

CHAPTER I.

1. HYDROSTATICS investigates the conditions fulfilled by the pressures produced by fluids when in a state of rest.

2. DEF. A FLUID is a material body which can be divided in any direction, and the parts of which can be moved among one another by any force however small.

[Water, Air, Gas, Mercury, Steam, are all instances of fluids.]

3. FLUIDS have been divided into *Elastic*, and *Non-elastic*.

(1) ELASTIC FLUIDS are those of which the dimensions are increased or diminished when the pressure upon them is increased or diminished.

[Air is an elastic fluid, as will hereafter be proved. So also is gas, and steam.]

(2) NON-ELASTIC FLUIDS are those of which the dimensions are not diminished by the addition of pressure, or increased by the withdrawal of pressure.

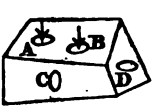
[Water, Mercury, and probably all other liquids, are compressible, but in a small degree only. The resistance, however, which they offer to compression is so great, that the conclusions obtained on the supposition of their being entirely incompressible are free from any sensible error, except in a very few cases.]

CHAPTER II.

PRESSURE OF NON-ELASTIC FLUIDS.

4. PROP. 1. FLUIDS press equally in all directions.

Let a close vessel of any shape be filled with fluid, and let *A*, *B*, *C* and *D*, similar and equal portions of the sides of the vessel, be removed, and their places supplied by plugs fitting the orifices exactly and acted upon by pressures just sufficient to keep them at rest. Then if an additional force *P* be applied to any one of the plugs, it is found that an *equal* additional pressure must be applied to *every other* of the plugs in order to prevent the fluid bursting out.



This experiment proves that a pressure communicated to the surface of a fluid at rest is transmitted by the fluid, undiminished and unimpaired, to

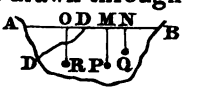
every other equal and similar portion of the surface the fluid is bounded by.

Also, whatever be the directions in which any of these plugs are inserted through the sides of the vessel, the same quantity of pressure is required to keep them at rest; which shews that, at the same point in a fluid, the fluid presses with equal force in *all* directions.

Wherefore, "Fluids press equally in all directions." Q. E. D.

5. PROP. II. **The pressure upon any particle of a fluid of uniform density is proportional to its depth below the surface of the fluid.**

Let the vertical lines *PM* and *QN* be drawn through *P* and *Q* two particles situated in the interior of a fluid of uniform density at rest, and let them meet the surface of the fluid in the points *M* and *N*.



Supposing the pressures on *P* and *Q* to be produced by the weight of fluid particles of the same magnitude as *P* and *Q* placed touching each other along the lines *PM* and *QN*, then, since the fluid is of uniform density, the weights of such lines of particles will be proportional to their lengths.

$\therefore$ Pressure on the particle *P*
: pressure on the particle *Q*
:: weight of the line of particles *PM*
: weight of the line of particles *QN*
:: *PM* : *QN*.

Again; the fluid being at rest, the pressure of any part of it will not be affected if we suppose the

particles (which are at rest) of the fluid upon which the pressure acts, to become connected with one another. Whence it will follow, that whether there be an uninterrupted line of fluid particles (as *RO*) reaching from a particle *R* to the surface of the fluid, or part of the line be cut off by a portion *ADD* of the fluid becoming solid, the pressure upon the particle *R* will still be the same.

Wherefore, "The pressure, &c." Q. E. D.

6. PROP. III. When a fluid is at rest its surface is horizontal.

[The manner in which the matter composing non-elastic fluids acts is twofold; first, by the particles pressing with their weights on those immediately below them; and second, by the property these particles possess of communicating, in all directions and without diminution, any pressure to which they are subjected.]

Imagine two contiguous particles *P* and *Q* situated in the same horizontal plane in a fluid at rest. Since the action of gravity on these particles is perpendicular to the plane in which they lie, its action on either of them can produce no effect in that plane. *P* and *Q* are therefore kept at rest entirely by the action of the surrounding fluid, and this action must be the same on each, or motion would ensue.

But since fluids press equally in *all* directions, and the horizontal pressures of *P* and *Q* on one another are equal, their vertical pressures, which are as their distances *Pa* and *Qb* below the surface of the fluid, must also be equal; and

$$\therefore Pa = Qb;$$

$\therefore$ *ab* is parallel to *PQ*, and is $\therefore$ horizontal.

And since in like manner the line joining any two adjacent particles in the surface of the fluid may be shewn to be horizontal, therefore the surface itself is horizontal.

Wherefore, "When a fluid, &c." Q. E. D.

7. PROP. IV. If a vessel, the bottom of which is horizontal and the sides vertical, be filled with fluid, the pressure on the bottom will be equal to the weight of the fluid.

The sides of the vessel being vertical, and the whole of the fluid being conceived to be made up of vertical straight lines of fluid particles, each of these lines will press vertically with its weight, and the sum of these vertical pressures will be the weight of the fluid in the vessel. Now the base of the vessel, being horizontal, will counterbalance and entirely destroy all the vertical pressures upon it. The pressure, therefore, sustained by the horizontal base of a vessel whose sides are vertical is equal to the weight of the fluid in the vessel. Q. E. D.

8. [From Propositions II and IV the following conclusion may be drawn.

The pressure exercised by a fluid on any horizontal plane placed in it, is equal to the weight of a column of the fluid whose base is the area of the plane, and whose height is the depth of the plane below the horizontal surface of the fluid.

For let *AB* and *CD* be two equal areas in the same horizontal plane; then since the pressure is the same on every point in the same horizontal plane, and (the areas of *AB* and *CD* being equal) the number of particles in the plane *CD* is the same as that in the plane *AB*, therefore the pressure on *AB* = pressure on *CD*.

But if a vertical line of fluid particles reach from every particle in *AB* to the surface, then, supposing the rest of the fluid to become solid, *EABF* becomes a vessel with vertical sides and horizontal base, and the pressure on *AB* = weight of the column *EB*.

Therefore pressure on *CD* = weight of a column of the fluid whose base is *CD*, and whose height is the depth of *CD* below the horizontal plane of the surface of the fluid in the vessel. Q. E. D.]

9. [From Art. 43 of the MECHANICS it appears that the pressure on the center of gravity of a system of bodies considered as heavy points is the same as if the weights of the bodies were collected at it. So long therefore as the quantity of matter in the system remains the same, the pressure produced by the system will be the same in whatever manner the several parts of the system may be arranged.

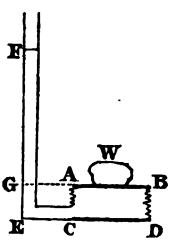
Now though at first sight it might appear probable that the pressure on the bottom or the sides of a vessel filled with fluid depended, somehow or other, on the quantity of the fluid by which the pressure is produced, such is by no means the fact,

but it is found, both from experience and by theory, that the pressures produced by fluids in a state of rest on the surfaces of the vessels containing them do not bear any uniform and definite relation whatever to the quantities (or the weights) of the fluids employed;—a conclusion so much at variance with the law governing the amount of pressure produced by solid bodies, that it has been termed THE HYDROSTATIC PARADOX.]

10. PROP. V. To explain the Hydrostatic Paradox.

THE HYDROSTATIC PARADOX is this,—that "any Pressure, however small, may by means of a fluid be made to counterbalance *any other* Pressure, however large; and whether the quantity of fluid employed be great or small; the same result can be effected."

The top and bottom of a vessel *AD* are boards, which are connected together by leathern sides. The vessel communicates with a vertical tube *EF* of small uniform bore, by means of a horizontal pipe *CE*. Let (*A*) be the area of the board *AB*, and (*a*) that of a horizontal section of the pipe *EF*.



Let *AB* be held in a horizontal position, and water be poured into the tube until it just rises in *AD* to *AB*. The water will therefore rise in the tube to *G*, a point in the same horizontal plane as *AB*; Prop. III.

If now there be a heavy weight W laid upon AB, it is found that it can be supported at rest by a small quantity FG of water poured into the tube. To shew the reason of this.

Since the sides of the tube FG are vertical, the pressure on the horizontal section of the tube at G is the weight of the fluid column FG; suppose w to be this weight.

Then since the pressures at every point in the same horizontal plane of a fluid at rest are the same,

$\therefore$ pressure on the area (A) of AB : pressure on the area a at $G :: A : a$,

$$\text{or} \quad W : w :: A : a;$$

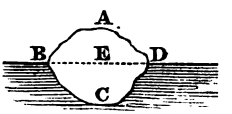
$$\therefore w = W . \frac{a}{A}.$$

If therefore W and A be given, w may be made as small as we please by diminishing a, which may be done to any extent.

Whether the pressure on the area a at G be produced by the weight of the fluid column GF, or by a plug acted on by any force, the pressures on the areas (a) at G and (A) at AB will still bear to one another the ratio of a to A,—a ratio wholly independent of the quantity of fluid contained in the vessel.

11. PROP. VI. If a body float on a fluid, it displaces as much of the fluid as is equal in weight to the weight of the body; and it presses downwards, and is pressed upwards, with a force equal to the weight of the fluid displaced.

Let *ABCD* be a body that displaces the portion *BEDC* of the fluid on which it floats. The pressure of the floating body on the fluid is produced by its weight, and it acts vertically downwards. The effective pressure therefore of the fluid, by which the body is kept at rest, must act vertically upwards, and be equal to the weight it balances.



Now let the floating body be removed, and the space *BEDC* be filled with fluid the same as that on which the body floated. The equilibrium of the fluid will not be disturbed; neither will the pressure of that part of it which was formerly in contact with the surface of the floating body be altered, if we suppose the parts of the fluid now in *BEDC* to become permanently connected with one another, and so to form a solid.

Suppose this to take place; then the pressure downwards of *BEDC* is its weight. And this being counteracted by the same sustaining power as that which balanced the weight of the floating body, it follows that the weight of the floating body is equal to the weight of the fluid it displaces.

Wherefore, "If a body float on a fluid, &c." Q. E. D.

CHAPTER III.

SPECIFIC GRAVITIES.

12. THE BULK, or VOLUME, or MAGNITUDE of a solid body is measured by the number of times the body is greater than some particular body previously fixed upon to compare the sizes of other bodies by, which is called "the unit of solid measurement."

A cube of which the edge is an inch long is called "a cubic inch." In the following pages a cubic inch will be taken for the unit of measurement, so that when it is said that M is the *bulk*, or *magnitude*, of a body, it is meant that the body contains M cubic inches.

13. DEF. The SPECIFIC GRAVITY of any substance is the weight of an unit of its magnitude.

If, as stated in the last Article, the magnitude of a body be measured by the number of *cubic inches* it contains, and the weight of a cubic inch of the substance the body is composed of be expressed in *grains*, then it will follow that if S be the Specific Gravity of any substance, S is the *number* of *grains* that *one cubic inch* of that substance *weighs*.*

* The Tables which are commonly called "TABLES OF SPECIFIC GRAVITIES" give the ratios which the weight of bulks of various substances bear to *equal* bulks of Water.

It

14. PROP. VII. If M be the Magnitude of a body, S its Specific Gravity, and W its Weight, $W = MS$.

It being found that the weight of a piece of Iron of any size : the weight of a bulk of Water of the same size :: 7·8 : 1, and the weight of a bulk of Silver : weight of an equal bulk of Water :: 10·5 : 1, and so for other substances,—Tables have been formed in which the numbers 1, 7·8, 10·5, &c. are placed opposite the words "Water" "Iron" "Silver" &c.; and by means of these tables, as we shall immediately shew, the weight of any bulk of any of the substances so registered can be determined, if only the weight be known of some particular magnitude (a cubic inch suppose) of some one of them.

The numbers given in these Tables are called, by some writers, the "Specific Gravities" of the substances they are attached to, but the enunciation of Prop. VII. Art. 14, will not permit of their being called so here.

"The Tables of Specific Gravities" give

Platina	21·5	Zinc	6·9
Gold	19·4	Sea Water	1·027
Mercury	13·6	Water	1·
Lead	11·4	Proof Spirit	0·917
Silver	10·5	Pure Alcohol	0·796
Copper	8·9	Air, at the surface of the Earth,—the average	0·00125
Iron	7·8		
Tin	7·3		

Before proceeding to point out the use of the Tables, it will be necessary to shew that "the weights of any two substances are in the ratio of the numbers given by the Tables as corresponding to the substances."

For w, w', w'' being the respective weights of equal bulks of Water, Iron, and Silver, since, as explained above,

$$w' : w :: 7{\cdot}8 : 1$$

$$\text{and } w : w'' :: 1 : 10{\cdot}5;$$

$\therefore$ compounding these proportions,

$$w' : w'' :: 7{\cdot}8 : 10{\cdot}5.$$

Suppose then it be required to find the weight of a cubic foot of Iron, the weight of 10 cubic inches of Silver being 61 ounces nearly.

Weight of a cubic foot (or $12 \times 12 \times 12$ cubic inches) of Silver,

$$= 12 \times 12 \times 12 \times \frac{61}{10} \text{ ounces};$$

$\therefore$ Weight

Let M = number of cubic inches in the body.

S = Specific Gravity of the body; i. e. let S be the number of grains that one cubic inch of it weighs.

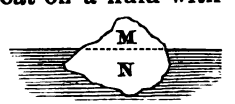
W = number of grains which the whole body weighs.

Then since $\overset{\text{grains}}{W} : \overset{\text{grains}}{S} :: \overset{\text{c. inches}}{M} : \overset{\text{c. inch}}{1}$

$\therefore \quad W = M \times S.$ Q. E. D.

15. PROP. VIII. When a body of uniform density floats on a fluid, the part immersed : the whole body :: the specific gravity of the body : the specific gravity of the fluid.

Let a solid of uniform density float on a fluid with M cubic inches of it above the horizontal plane of the surface, and N cubic inches below that surface.



Let S = S. G. (Specific Gravity) of the solid; S' = S. G. of the fluid.

Then $(M + N) \times S$ = weight of the solid. Prop. VII.

$N \times S'$ = ...fluid displaced.

But because the body floats these two weights are equal; by Prop. VI.

$\therefore$ Weight of a cubic foot of Iron

$$= \frac{12 \times 12 \times 12 \times 61}{10} \times \frac{7{\cdot}8}{10{\cdot}5}$$

$$= \frac{12 \times 12 \times 12 \times 61 \times 7{\cdot}8}{105} = 7830 \text{ ounces, nearly.}$$

$$\therefore\ N.S' = (M+N).S;\quad \text{and } \therefore\ \frac{N}{M+N} = \frac{S}{S'},$$

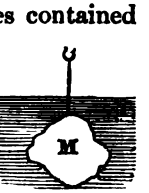
$$\text{or}\quad N : M+N :: S : S';$$

That is,

The part of it immersed : the whole of the body :: S. G. of the body : S. G. of the fluid. Q. E. D.

16. PROP. IX. When a body is *immersed* in a fluid, the weight lost : whole weight of the body :: the specific gravity of the fluid : the specific gravity of the body.

Let M be the number of cubic inches contained in a body of uniform density which is wholly immersed in a fluid; S the S. G. of the body, S' the S. G. of the fluid.



Then the pressure downwards of the solid is its weight, and if the solid be removed, and the space it filled be occupied by an equal bulk of the fluid, equilibrium will still remain. And if the fluid so added become solid, the equilibrium will continue to exist, and the pressure upwards of the surrounding fluid will remain the same as before.

Now under this supposition the pressure downwards of the fluid added is its weight. And since the pressure upwards of the surrounding fluid supports this weight, it must be exactly equal and opposite to it.

The pressure downwards therefore of the original solid, (i. e. its weight MS), must have been diminished by a pressure upwards arising from the

surrounding fluid exactly equalling in magnitude the weight of the fluid displaced, which is by Prop. VII equal to MS'.

$\therefore$ Weight lost by the body : the whole weight of the body :: $MS : MS'$

:: $S : S'$

:: S. G. of body : S. G. of fluid. Q. E. D.

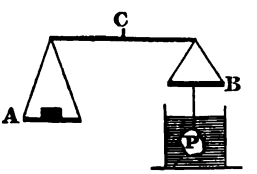
17. [It appears from the proof of the last Proposition, that the pressure of a fluid on a body wholly immersed in it acts vertically upwards, and is equal to the weight of the fluid which the body displaces.

If this pressure be less than the weight of the body,—that is, if the S. G. of the fluid be less than that of the solid,—the pressure downwards arising from the weight of the solid will be greater than the pressure of the surrounding fluid upwards, and the body will therefore sink to the bottom of the vessel.

But if the pressure of the fluid upwards be greater than the weight of the body immersed,—that is, if the S. G. of the fluid be greater than that of the solid,—the pressure upwards will be greater than the pressure downwards, and the body will therefore rise until the conditions of Proposition VIII are fulfilled,—when it will float.]

18. PROP. X. (1) To describe the Hydrostatic Balance; and (2) to shew how the Specific Gravity of a body may be found by it,—*first*, when its Specific Gravity is *greater* than that of the fluid in which it is weighed,—and *second*, when it is *less*.

(1) The Hydrostatic Balance is the common balance with a hook attached to the under part of one of its scales, so that bodies may be weighed either by putting them into the scale, or by suspending them from it and letting them be immersed in a fluid, as here represented.



(2) FIRST, let the S. G. (Specific Gravity) of the body be *greater* than that of the fluid.

Since the S. G. of the solid is greater than that of the fluid, the body will sink in the fluid. Prop. IX. Cor.

Let S = S. G. of the solid, S' = S. G. of the fluid.

W = weight required to balance the body when placed *in* the scale.

W' = weight required when the body is immersed.

$\therefore$ $W - W'$ = diminution of the body's weight in consequence of immersion.

Therefore, by Prop. IX.,

$$W : W - W' :: S : S';$$

$$\therefore S = \frac{WS'}{W - W'}.$$

Whence, W and W' being known, S may be determined if S' be given.

NEXT, let the S. G. of the body be *less* than the S. G. of the fluid.

When the body, in this case, is forced under the surface of the fluid, the pressure downwards on

it (which is the weight W of the body) being less than the pressure upwards (which is the weight of a quantity of fluid equal in magnitude to the fluid displaced) the body must on the whole be acted on by a pressure upwards, which is equal to the weight of the fluid displaced *minus* the weight of the solid.

Let a body Q be taken, of greater S. G. than that of the fluid, and large enough to sink both itself and the body P whose S. G. is required when P is attached to it. Let Q be immersed in the fluid and balanced by weights in A. Next, leaving the weights in A, let P be attached to Q and both of them be immersed. The scale A will now preponderate, and to restore the equilibrium a portion (W') of the weights in it must be removed. We have now therefore, (W being the weight of the body P when placed *in* the scale B),

W' = whole effective pressure on P when immersed; which acts upwards;

= weight of an equal bulk of fluid *minus* weight of P (W;)

$\therefore$ weight of a bulk of fluid equal to that of $P = W + W'$.

Now, since by Prop. VII. $W = MS$, when the magnitudes of the bodies are the same, $W \propto S$;

$$\therefore W : W + W' :: \text{S. G. of solid} : \text{S. G. of fluid}$$

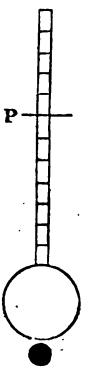
$$:: S : S';$$

$$\therefore S = \frac{WS'}{W + W'}.$$

Whence,—W and W' being known weights,—S can be found if S' be given.

19. PROP. XI. (1) To describe the common Hydrometer, and (2) to shew how to compare the Specific Gravities of two fluids by it.

(1) The common Hydrometer consists of two spheres touching each other, and a long uniform slender stem, which if produced, would pass through the centers of both of them. The upper sphere is hollow, and the lower is filled with small shot or mercury, to serve as ballast for the instrument and to make it float steadily in a vertical position when put into a fluid. The stem is graduated by divisions of the same length.



(2) The instrument is made lighter than equal bulks of the fluids whose Specific Gravities it is used to compare. Suppose the bulk of the portion of the stem included between every two graduations to be one four thousandth part of the bulk of the whole instrument. When the Hydrometer floats on a fluid whose S. G. is S, let 20 divisions be above the surface; and when it floats on a fluid whose S. G. is S', let there be 30 divisions out.

Now the weights displaced of the fluids are in both cases the same, namely the weight of the instrument; Prop. VI. If M and M' therefore be the magnitudes of the fluids displaced, we have, by Prop. VII,

MS = weight of the Hydrometer = $M'S'$;

$\therefore S : S' :: M' : M$

$:: 4000 - 30 : 4000 - 20$

$:: 3970 : 3980$;

and the ratio of the S. G.'s of the two fluids is determined.

[A mark *P* is made at the point in the stem to which the instrument sinks in a fluid called "Proof Spirit", which is a mixture consisting of equal *weights*,—(*not* equal *magnitudes*),—of pure Alcohol and of Water. Alcohol being lighter than Water, if a mixture of these two fluids contain a greater weight of the former than it does of the latter, it will be lighter than Proof Spirit, and the Hydrometer therefore will displace a greater bulk of it than it does of Proof Spirit. Wherefore the surface of such a mixture will rise to a higher point in the stem than *P*, and in that case the mixture is said to be "*above proof*".

On the other hand, if the weight of the Water contained in the mixture be greater than that of the pure Alcohol, the Hydrometer will not sink so low as to the point *P*, and the fluid is then said to be "*below proof*".

When equal weights of pure Alcohol and of Water are mixed, the magnitudes of the fluids are nearly as 3 to 2;—so that out of every hundred cubic inches of Proof Spirit about sixty are pure Alcohol and forty are Water.]

CHAPTER III.

ELASTIC FLUIDS.

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20. Prop. xii. Air has Weight.

This is proved by these experiments.

If a close vessel be weighed after the air has been exhausted from it, and afterwards when the air has been admitted into it, it is found that the weight is greater in the latter than in the former case.

If a bladder be weighed in a vessel from which all the air has been exhausted by a method which will be described hereafter, first, when the bladder contains no air, and again after air has been forced into it, the weight required to balance the bladder is greater in the latter than in the former case.

Whence we conclude that 'Air has weight.'

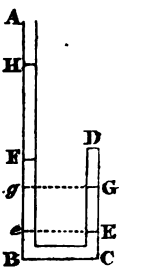
21. Prop. xiii. The elastic force of Air at a given temperature varies as the density.

This is proved by experiment.

Let *ABCD* be a tube, of which the bore is uniform (its section being an area *A*), and the legs *AB* and *CD* are vertical. The end *A* of the tube is open, and *D* is closed.

A quantity of air is retained in the shorter leg by some mercury which rises to *E* and *F* in the two legs respectively. Through *E* the horizontal line *Ee* is drawn.

Then, The weight of the mercurial column Fe
= pressure *downwards* at e on a surface A; by Prop. IV.
= ..... *upwards* at e ..... ; Prop. I.
= ..... *upwards* at E ..... ; Prop. II.
= pressure by the air in DE on the surface A of mercury with which it is in contact; (because, the whole being at rest, the pressures upwards and downwards on the same horizontal plane must be equal.)



Similarly, if when more mercury is poured in, its surfaces stand at G and H in the two legs, we have, drawing Gg horizontally through G,

Weight of the mercurial column Hg = pressure by the air in DG on the surface A of mercury with which it is in contact.

Now if ED, Fe, GD, Hg be measured, it is invariably found, however the quantities of air and of mercury used in the experiment be varied, that

$$DE : DG :: Hg : Fe;$$

$\therefore$ content of DE : content of DG :: content of Hg : content of Fe.

But since the same quantity of air is contained in DE and in DG,

$\therefore$ Density of the air in DE : density of the air in DG
:: content of DG : content of DE
:: content of Fe : content of Hg
:: Weight of mercury in Fe : weight of mercury in Hg,
:: Pressure of air in DE on surface A : pressure of air in DG on same surface;—as shewn above.

Whence we conclude that

The pressure which air exercises on a given surface varies as the density of the air. Q. E. D.

[NOTE. In the experiment here described the longer leg of the tube is placed in a vessel from which the air has been exhausted, so that there is no pressure at all in the surface of the mercury in the longer leg. If *A* communicated with the open air, the surrounding atmosphere would press on the surface of the mercury in *AB* and would have to be taken into consideration in estimating the whole amount of pressure sustained by an area *A* placed at *e* or *g*.

The method of making the experiment described in the above Proposition has been selected as being more simple than the other. The same result, however, is arrived at in whichever of these two ways the experiment is conducted.]

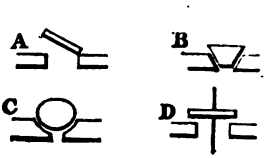
22. PROP. XIV. The elastic force of the air is increased by an increase of temperature.

This is proved by experiment.

If a bladder be partially filled with air and then brought near the fire, the included air immediately expands and the bladder becomes fully distended. On the included air cooling down, the bladder returns to its original flaccid appearance.

23. DEFS. (1). A VALVE is a kind of door which fits an orifice, such that if a fluid press on one side of it, it opens and allows the fluid to pass through, but which keeps the orifice tightly closed if the fluid press on the other side.

[Valves are of various forms,—a flap of leather (*A*) fastened at one edge,—a frustum of a cone (*B*) made of metal,—a sphere (*C*),—or a plate of metal (*D*) with an axis passing perpendicularly through it. By any of these contrivances the flow of a fluid upwards would be



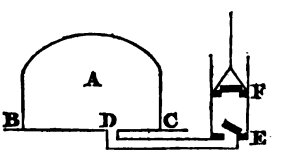
prevented only by the weight of the valve, but a rush of fluid from above would carry the valve along with it, and keep the orifice which the valve fits completely closed.

When the fluids employed are very rare,—like air or gas,—the valves are generally made of flaps of oiled or varnished silk, which, being attached at two or three points to the surfaces in which the orifices are situated, are raised by a very slight pressure, and so allow a fluid to pass under whose density is exceedingly small.]

(2) A PISTON is a plug, mostly cylindrical, which, by means of a rod attached to it, can be made to play within a tube which it fits accurately.

24. PROP. XV. To describe the construction of the common Air Pump and its operation.

DESCRIPTION. The Air Pump consists of a glass vessel *A*, called the *Receiver*, made to fit a table *BC* so as to be air tight. A tube *DE* connects the Receiver with a cylinder *EF*, called the *Barrel*. At the bottom of the Barrel there is a valve *E* opening upwards, and a piston *F* (also furnished with a valve opening upwards) plays within the Barrel.



OPERATION. Let the piston *F* be at its highest point, and the instrument filled with air the same as that of the surrounding atmosphere.

When the piston is driven down, the air at first in the barrel is condensed and its pressure therefore

increased. The valve *E* is kept closed, and the valve in *F* being pressed on the under surface more strongly than on the upper, opens and allows the air in the barrel to escape through it.

Next, on raising the piston, the external air keeps *F* closed, and there now being no air in *EF*, the pressure on the under surface of the valve at *E* will open that valve and allow air from the receiver and pipe to flow into the barrel. The figure represents the instrument during the ascent of the piston.

When the piston descends again, the air in *EF* is condensed until it opens the valve in *F* and escapes; and on the piston reascending, more air rushes from the receiver into the barrel.

The process may be repeated until the air in the receiver becomes so rare, that its pressure is insufficient to overcome the weight of the valve at *E*.

25. PROP. XVI. To describe the construction of the Condenser and its operation.

CONSTRUCTION. The Condenser is a *Barrel AB* furnished with a piston *A* that has a valve in it opening downwards; at the bottom of the Barrel there is a like valve *C*. The neck of the Barrel communicates with an air-tight vessel *D*, called the *Receiver*.

OPERATION. Let the instrument be filled with common air, and the piston be at its greatest height. On the piston being forced down, the air in the barrel is condensed, and its pressure being therefore increased;

Prop. XIII, it keeps the valve *A* closed, opens the valve *C*, and escapes into the Receiver. The figure represents the instrument during the descent of the piston.

On the piston's ascending, the air in the receiver keeps *C* closed, and there now being no pressure on the under surface of the valve *A*, the external air opens the valve, and the barrel is filled with air, which may be driven into the receiver by forcing down the piston a second time. In this manner the condensation of the air in the receiver may be carried on to any required extent.

[The communication between the barrel and the receiver can be cut off at pleasure by means of a stop-cock at *E*, and the barrel is made to screw off and on at its lower end.]

26. PROP. XVII. **To explain the construction of the common Barometer, and to shew that the mercury is sustained in it by the pressure of the air on the surface of the mercury in the basin.**

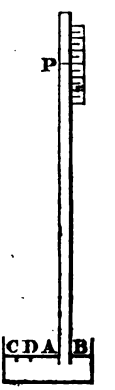
The Barometer is an instrument for measuring the pressure of the air. It consists of a glass tube (see fig. Prop. XVIII.) of uniform bore, closed at one end, and not less than 33 or 34 inches long. This tube is filled with mercury, and the open end, being first stopped with the finger, is placed below the surface of some mercury in a basin, when the tube being held in a vertical position and the finger being withdrawn, the mercury in the tube subsides, and stands at a height above the mercury in the basin

that varies on different days from about 28 to 32 inches. A small scale of inches is attached to the upper part of the tube, to shew the height at which the mercury may be standing at any time.

That the column of mercury is supported in the tube by the pressure of the atmosphere appears from this experiment. When the whole is put into the receiver of an air pump, the mercury sinks in the tube more and more for every barrel of air that is pumped out, until at length it is all but on a level with the surface of the mercury in the basin. On readmitting the air into the receiver the mercury in the tube rises again to its first level.

27. PROP. XVIII. **The pressure of the atmosphere is accurately measured by the weight of the column of mercury in the [vertical tube of the] Barometer.**

Let P be the surface of the mercury in the vertical tube; AB the section of the tube made by the horizontal surface of the mercury in the basin; CD an area equal to AB in that surface.



Then, Weight of mercury contained in PB

= pressure downwards on the area AB, Prop. IV;

= pressure upwards on AB,

= pressure upwards on CD, since AB and CD are equal areas situated in the same horizontal plane of a fluid at rest. Prop. II.

But the fluid being at rest the pressure *downwards* on CD, which arises from the pressure of the air in contact with it, is equal to the pressure upwards on it.

Wherefore the pressure of the atmosphere on any area CD is equal to the weight of the column of mercury supported in the tube of the barometer, whose base AB is equal to that area.

Wherefore, "The Pressure, &c." Q. E. D.

[Cor. 1. By Art. 8. the pressure of a fluid on a horizontal plane immersed in it is the weight of a column of the fluid whose base is equal to the area of the plane and whose height is the depth of the plane below the surface of the fluid. Wherefore the pressure exerted by the atmosphere on such an area,—being measured by the weight of the column of mercury (of equal section) that is supported in the barometer,—is equal to the weight of a column of mercury whose base is equal to the plane acted upon and whose height is the same as that of the column of mercury supported in the vertical tube of the barometer.]

[Cor. 2. The pressure of the air being measured by the weight of the column of fluid which it supports, the Barometer might be filled with any other fluid quite as well as with mercury. But mercury being by far the heaviest fluid known, the column of mercury required to produce a given pressure is very much shorter than if any other fluid were employed.

For example, if it took 30 inches of mercury to balance the pressure of the air, then since mercury is 13·6 times heavier than water (see note p. 57.), it

would take a column of water $13{\cdot}6 \times 30$ inches, or 34 feet high, to produce the same effect.]

[Cor. 3. The mercury in the barometer standing at 30 inches, the pressure of the air on a surface whose area is a square inch is equal to the weight of a column of mercury 30 inches long and whose base is a square inch, i.e. to the weight of 30×1, or 30, cubic inches of Mercury.—*To find how much this pressure amounts to.*

The weight of a cubic foot of Water is found to be 1000 ounces avoirdupois;

$\therefore$ the weight of a cubic foot of Mercury $= 1000 \times 13{\cdot}6$ oz. (See page 57. Note.)

$\therefore$ $\overset{\text{cubic foot}}{1}$, or $\overset{\text{cubic inches}}{12 \times 12 \times 12}$, : $\overset{\text{cubic inches}}{30}$ :: $\overset{\text{oz.}}{1000 \times 13{\cdot}6}$: weight of 30 cubic inches of mercury, which $\therefore = \dfrac{13600 \times 30}{12 \times 12 \times 12}$ ounces = nearly 236 oz. or $14\frac{3}{4}$ lbs.]

[Cor. 4. In Chapters I and II the pressure of the air on the surface of the fluid contained in an open vessel has been left out of consideration; or, more properly speaking, the experiments described in those Chapters have been supposed to be made in the exhausted receiver of an air-pump.

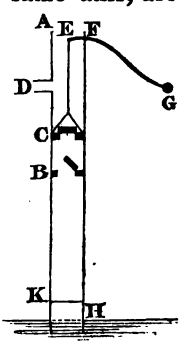
From Cor. 2 it appears, that when Water is the fluid employed, the pressure of the air on its surface is equivalent, (when the mercurial barometer stands at 30 inches), to that which would be produced by a head of water 34 feet high. In estimating, therefore, the pressure on any surface placed at a given depth below the surface of water, this large additional pressure, (amounting by Cor. 3 to more than

14 lbs. on every square inch of surface), must not be left out of the account.

For suppose two surfaces *A* and *B*, each a square inch in area, to be placed horizontally at the respective depths of 2 and 6 feet below the surface of some water at rest. If there be no pressure of air on the horizontal surface of the water, the pressures on *A* and *B* will be as 2 : 6, or as 1 : 3. But if the height of the mercury in the barometer be 30 inches, and the atmosphere is in contact with the surface of the water, then the pressures on *A* and *B* will not only be greatly increased, but the ratio they bear to each other will be materially altered; the pressure on *A* now becoming to that on *B* as 34 + 2 : 34 + 6, that is as 36 : 40, or as 9 : 10, instead of being, as it was before, as 1 : 3.]

28. PROP. XIX. To describe the construction of the Common Pump and its Operation.

CONSTRUCTION. In the *Common Pump* two hollow cylinders, *AB* and *BH*, having the same axis, are connected together, and at their junction a valve *B* opening upwards is placed. The higher *AB* of these cylinders is called "*the Body of the Pump*"; in which a piston *C*, containing a valve opening upwards, plays by means of a rod attached to the end *E* of a lever *EFG* whose fulcrum is *F*. A spout *D* is placed just above the highest point to which this piston ascends. The lower cylinder *CH*, which is called "the *Suction Pipe*", reaches below the surface *H* of a well of water.



OPERATION. Let the piston be at *B* and the suction tube full of air. As *C* is raised, the pressure of the external air keeps the valve in *C* closed, and a vacuum being consequently made in the lower part of *CB*, the air in *HB*, pressing against the under surface of the valve at *B*, opens it, and escapes into *BC*.

The air therefore, which before the ascent of the piston occupied the space *BH*, now occupies the greater space *CBH*, and so becoming less dense than before, it exercises a less pressure on the surface of the water at *H*; Prop. XIII. Wherefore, since the external air continues to exercise the same pressure as before on the surface of the water in the well, it will force up water into the suction pipe to a height *K* such that the pressure of the air in *CK*, together with the pressure arising from the column of water *KH*, produces the same effect on the section of the water in the suction tube at *H*, as the external air does on an equal area in the surface of the water in the well.

When *C* has come to the highest point of its ascent, and equilibrium exists between the pressure of the external air on the surface of the water in the well and the pressure of the air in *CK* together with that arising from the fluid column *KH*, the valve *B*, being equally pressed on its upper and its under surfaces, will fall down by its own weight. The piston *C* is then forced down; the air in *CB* is condensed until its pressure becomes greater than that of the external air, when it opens the valve in *C* and escapes.

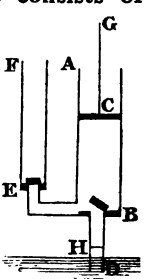
When *C* is raised again, the same circumstances recur. The water rises a little higher in the suction pipe at every stroke of the piston, and at last flows through *B*, when, on *C* descending again, it gets above the valve *C*, and is brought up to the spout at *D*.

[As the average pressure of the atmosphere will not support a column of water more than 34 feet high, if the valve *B* be more than 34 feet above the surface of the water in the well, the external air in that case not being able to force the water up so high as *B*, the pump will not work.

N.B. The figure represents the pump during an *ascent* of the piston, when (air, or water, flowing through it) the valve at *B* is *open*, and that at *C* is *shut*.]

29. Prop. xx. To describe the construction of the Forcing Pump, and its operation.

Construction. The Forcing Pump consists of a *Barrel AB* in which a solid piston *C* works by means of a rod *CG*; *BD* is a *Suction pipe* reaching below the surface *D* of a well of water; *BE* a pipe connecting *BC* with a cylinder *EF*; at *B* and *E* valves are placed, opening upwards.



Operation. Suppose the piston *C* to be at its greatest height, and the pump full of air. When *C* descends, the air in *AB* is condensed, and its pressure being increased (Prop. xiii.), the valve *E* is opened, and the air at first in *AB* is forced into *EF*.

On the piston reascending, the air in *EBC*, being now less dense than that in *EF* and *BD*, the valve *E* is kept closed by the external air, the valve *B* is opened, and a portion of the air at first in *DB* rushes into the Barrel. The external air then forces some water a little way up the pipe, (to *H* suppose), until the pressure on the horizontal section of the suction pipe at *D*, (which arises from the elasticity of the rarified air in *HBC* together with the pressure of the column of water *HD*), is equal to the pressure of the external air on an equal area in the surface of the water in the well. When this has taken place, and there is equilibrium, the valve *B*, being pressed equally on both its surfaces, closes by its own weight.

The piston again descends, and the air in *AB* is driven through *E* into *EF*; it ascends again, and more water is forced up the Suction pipe; and these operations are repeated until the water rises above *B*, when on the next descent of the piston the water above *B* is driven into *EF*.

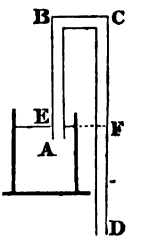
[N. B. The Figure represents the Forcing Pump during the *ascent* of the piston, when the valve *E* is *shut*, and the air (or water) is rushing from the Suction pipe into the Barrel through the *open* valve *B*.]

30. Prop. xxi. To explain the action of the Siphon.

The Siphon is an instrument for drawing fluids out of vessels; it is a bent pipe or tube, with legs of unequal lengths.

Suppose the Siphon *ABCD* to be of uniform bore,—the two legs *AB* and *CD* parallel and con-

nected together by a pipe *BC* at right angles to them. Let the instrument, after it is filled with the same fluid as that in the vessel and its ends are stopped, be placed with the legs vertical, and the end *A* of the *shorter* leg plunged below the surface *E* of the fluid in the vessel. Let *a* be the area of the section of the pipe and let *F* be in the same horizontal line as *E*. Suppose the ends *A* and *D* to be opened, and thin films of fluid to fit the pipe at *E* and *D*.



Then, the pressure upwards on the film at *D* = pressure of the atmosphere on the area *a*; and the pressure downwards on it = weight of the fluid-column *CD*.

Similarly, the pressure upwards on the film at *E* = pressure of the atmosphere on an equal area *a* in the surface of the fluid in the vessel; and the pressure downwards on it = weight of the column *BE* of fluid.

Wherefore, the whole pressure on the column *EBCD* of fluid is, in the direction *EBCD*, the pressure of the atmosphere on an area *a* + weight of column *CD* of fluid; and the whole pressure on the same column in the direction *DCBE*, is the pressure of the atmosphere on an area *a* + weight of a column *BE* of fluid.* Wherefore, the pressure on the difference of these being a pressure on the fluid in the siphon in the direction *EBCD*, motion must

* The weight of the fluid in *BC* produces no pressure on the vertical columns *BA* and *CD*, but is supported by the under surface of the pipe *BC*; the fluid in *BC*, however, serves to transmit the vertical pressures in *AB* and *DC* from the one pipe to the other.

ensue, and the fluid must be forced out at *D*. And as fast as the fluid flows out, the pressure of the atmosphere will force up more from the vessel into the siphon, and so the motion will be continued.

[NOTE. If *Water* be the fluid employed, and *EB* be greater than 34 feet, the pressure of the column *BE* on the surface *a* placed at *E* will be greater than the pressure upwards on that surface which arises from the pressure of the atmosphere on the surface of the water in the vessel. When therefore the end *A* of the siphon is opened, the column *BE* will begin to sink down until its extremity stands at such a height above *E* in the pipe, as that the weight of the column of water in the tube shall just be balanced by the pressure of the atmosphere. If *D* be now opened, the water in *CD* runs out at *D*, and, no water flowing over *B*, the air will rush up the pipe *DC* and press on the upper end of the column of fluid supported in *BE*. This column therefore, being now equally pressed by the atmosphere at each of its extremities, will descend by its own weight into the vessel. THE SIPHON *therefore will not act, (when* WATER *is the fluid used,) if EB be greater than* 34 *feet.*]

31. PROP. XXII. To graduate a common Thermometer.

THE THERMOMETER is an instrument for comparing the intensities of the heats of solids or fluids. [See Figure to Prop. XXIII.]

It consists of a slender uniform tube *BA*, closed at the upper end, and terminating at the lower in a bulb. The bulb and part of the tube are filled with Mercury, Spirits of Wine, or any other fluid

(not of a gaseous form) which expands on being heated, and contracts with cold. If the fluid in the thermometer occupy the same space when the instrument is plunged into two different fluids, the temperature of those fluids will be the same.

To graduate a thermometer, plunge the bulb into *melting snow* and make a mark *A* at the point to which the mercury contracts. Next plunge the bulb into *boiling water* and make another mark *B* at the point to which the mercury rises.* The distance *AB* may then be graduated by equal divisions, and the graduation may be continued above *B* and below *A*, if it be required to compare temperatures by the instrument which are greater than that of boiling water and less than that of melting snow.

32. PROP. XXIII. Having given the number of degrees on *Fahrenheit's* thermometer, to find the corresponding number in the *Centigrade* thermometer.

In *Fahrenheit's* graduation of the scale of the thermometer, (which is that generally made use of in England), 32 is the number placed opposite to the *freezing point A*, and *AB* being divided into 180 parts (or *degrees*) the number placed opposite to the *boiling point B* is 32 + 180, or 212. The graduation of Fahrenheit therefore begins from a point in the scale which is below *A*.

* The temperatures of *boiling water* and of *melting snow* are taken to determine the fixed points *B* and *A* in the scale of the thermometer, because it is found, by experiment, that these temperatures are *fixed* and *invariable*; the mercury *always* falling to *A* when plunged into melting snow, whether the snow be melting quickly or slowly,—and *always* rising to *B* when plunged into boiling water, whether the fire applied to the vessel containing the water be great or small.

In the *Centigrade* thermometer, (which is that generally used on the Continent), the graduation begins from A, and AB is divided into 100 parts, or *degrees*.

Now let there be a thermometer furnished on one side with a scale graduated according to *Fahrenheit*, and on the other with a *Centigrade* scale. Let the mercury rise to any point D, and let F and C be the number of degrees respectively marked opposite to D on the scales of Fahrenheit and the Centigrade.

Now since the tube is uniform,

The number of degrees of Fahrenheit in AB, (180),

: ..... AD, $(F-32)$,

:: ..... of Centigrade... AB, (100),

: ..... AD, (C);

$$\therefore 180 \times C = 100 \times (F - 32)$$

$$\therefore C = \frac{100}{180} \cdot (F - 32) = \frac{5}{9} \cdot (F - 32).$$

Ex. If the mercury in Fahrenheit's thermometer stand at 77, then the correspondent number of degrees on the Centigrade will be

$$\frac{5}{9} \cdot (77 - 32) = \frac{5}{9} \cdot 45 = 25.$$